STEADY STATE ANALYSIS OF INDUCTION MOTOR FED BY VOLTAGE SOURCE INVERTER

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By
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TO

MY PARENTS

WITH

DEEP AFFECTION

AND

GRATITUDE

CERTIFICATE

This is to certify that this work entitled, "STEADY STATE ANALYSIS OF INDUCTION MOTOR FED BY VOLTAGE SOURCE INVERTER" submitted by Nirmal Dutta in partial fulfilment of the degree of Master of Technology to the Department of Electrical Engineering, Indian Institute of Technology Kanpur is a record of project work by him under my supervision and guidance. The results embodied in this thesis have not been submitted to any other University or Institute for the award of any degree.

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-Nirmal Dutta

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LIST OF PRINCIPAL SYMBOLS

I_1, I'_2	:	r.m.s. value of stator and r.m.s. value of rotor current referred to stator
i_A, i_B, i_C	:	rotor phase currents
i_a, i_b, i_c	:	stator phase currents
i_{1d}, i_{1q}	:	D and Q-axis instantaneous stator current
i'_{2d}, i'_{2q}	:	D and Q-axis instantaneous rotor current referred to stator
$\bar{I}_{1d}, \bar{I}_{1q}$	:	D and Q-axis stator current phasors
$\bar{I}'_{2d}, \bar{I}'_{2q}$	:	D and Q-axis rotor current phasors referred to stator
M	:	Mutual inductance between stator and rotor phase referred to stator
M_c	:	torque constant
m_i	:	modulation index
P	:	number of poles
p	:	differential operator (d/dt) or number of pulses/half cycle
p_1	:	instantaneous input power
$R_{c/f}$	:	ratio of carrier to inverter operating frequency
r_1, r'_2	:	stator winding resistance per phase and rotor winding resistance per phase referred to stator
S	:	normal slip
S_n	:	slip due to nth harmonic
T	:	fundamental time period over one cycle
t_e	:	instantaneous electromagnetic torque
t_{sh}	:	instantaneous steady harmonic torque

t_{ph}	:	instantaneous pulsating harmonic torque
v_a, v_b, v_c	:	stator phase voltages
v_A, v_B, v_C	:	rotor phase voltages
$v_{1d}, v_{1q},$	:	D and Q-axis stator voltages
v_{2d}, v_{2q}	:	D and Q-axis rotor voltages
V_{dc}	:	constant d.c. input voltage
V_1	:	r.m.s. value of stator phase voltage
ω_e	:	arbitrary electrical angular velocity
ω_r	:	rotor angular speed
ω_s	:	output angular frequency of the inverter
α_k, β_k	:	firing and extinction angles for kth pulse
θ_s	:	angular displacement between rotor phase A and the Q-axis
θ_2	:	angular displacement between stator and rotor axis
$\lambda_A, \lambda_B, \lambda_C$	:	rotor flux linkages
$\lambda_a, \lambda_b, \lambda_c$	:	stator flux linkages.

ABSTRACT

The present work deals with steady state analysis of current and torque harmonics of induction motor fed by three phase voltage source inverter.

A method of analysing constant speed performance of symmetrical induction machine fed with nonsinusoidal voltages is presented. It is based on Fourier analysis of impressed voltage waveforms. Equivalent circuits for various harmonics are derived from machine equations which were established for stationary D/Q reference frame attached to the stator. Stator current and torque harmonics and some useful expressions have been derived for various performance parameters through frequency domain analysis.

Emphasis has been given to each predominant harmonics for their individual effect to motor performance characteristics. These harmonic currents result in additional and sometimes rather large losses. A method of predicting pulsating harmonic electromagnetic torque includes the effect of harmonic variation in motor slip and inverter operating frequency. Simulated instantaneous waveforms for stator current and pulsating harmonic torque in the machine have been plotted on the computer. Several speed versus average torque and speed versus pulsating harmonic torque curves have been computed using the expression derived.

The next part of the thesis deals with the control of torque harmonics by modulating motor input voltage within the inverter. The use of E.P.W.M. technique applied to thyristor inverter to obtain adjustable voltage and frequency is reviewed. Only the ideally modulated inverter output voltage is considered to calculate voltage, current and torque harmonics. A theoretical study or the effect of changes in inverter frequency, modulation index and number of pulses per half cycle on machine performance characteristics have been carried out.

Chapter - 1

INTRODUCTION

1.1 GENERAL DISCUSSION

The advent of SCRs has revolutionized the field of speed control of electrical machinary. The solid state speed control system has many advantages over the older schemes which use motor generator sets, magnetic controller and gas discharge valves such as (i) reliability, (ii) fast acting, (iii) long life, (iv) less maintenance, (v) high efficiency and (vi) low cost.

A.C. and D.C. machines are the competitors in the industrial drive field. Though d.c. machine and low frequency a.c. commutator motor have dominated the field of variable speed electric drive because of their inherent speed/torque characteristics and simplicity of control, but still a.c. squirrel cage induction motor is used due to its inherent constant speed when operated from constant power supply. In addition, d.c. machines need careful maintenance and are not suitable when environment contains inflammable gases. Though the control module of a.c. drives involves complicated circuitry and the costs are comparatively higher, often the economy is realized through the lower price of the a.c. machines. The rapid increase in SCR's power handling capacity has made feasible to build converters, inverters, cycloconverters which are compact, reliable and efficient. These

have revived interest in the possibility of using squirrel cage induction motor due to its versatility, simplicity, ruggedness, less maintenance, high speed and voltage limit and high power to weight ratio.

Due to complexity of the problem encountered by the switching elements, exact analysis and design of many of the motor control systems is still under development. Output currents or voltage from these switching circuits are non-sinusoidal and the circuit voltages are difficult to determine in many instants. In this connection modern digital computers have come to the help of designers and researchers to a great extent. Computer based analysis and design have been developed for many complex systems. Although conventional models of the induction machine have been extensively used but many attempts have been made to develop models of the machine specially suitable for the application using thyristors.

1.2 METHODS OF SPEED CONTROL OF INDUCTION MOTOR

The various techniques of speed control of induction motors are:

(i) Stator voltage control

(ii) Stator voltage and frequency control

(iii) Rotor power control

(iv) Rotor impedence control.

(i) Stator Voltage Control: This is the simplest and most straightforward method for obtaining developed torque at different speeds both for squirrel cage and slip ring induction motor by adjusting the stator voltages using triacs or thyristors in various configuration [1, 2, 3]. Though for parabolic speed/torque characteristics like that of pumps, fans, the control scheme suits but for constant load torque speed range is limited. Besides this scheme is inefficient at low speeds because slip times the air gap power goes as heat loss in rotor as well as pull out torque decreases as the input voltage is reduced. Therefore this method of control is employed for small machines where economy and not efficiency is the prime consideration .

(ii) Stator Voltage and Frequency Control: For economic consideration, the magnetic circuit of the machine is designed such that the operating point is near the knee of the magnetization curve. When the frequency of stator excitation is varied, the r.m.s. value of the stator voltage must also be adjusted to keep the air gap flux of the machine to be constant which is the most efficient method of speed control of squirrel cage induction motor. Normally a variable d.c. supply is obtained using controlled rectifiers [1, 2, 3] and the d.c. supply is fed to an inverter which applies a variable frequency supply to the machine. With the help of suitable feedback to control the rotor slip and stator excitation it is possible to have constant torque or constant power operation.

Variable frequency can also be obtained using cycloconverter [4]. Since there is no d.c. link in it, these are more efficient than inverter fed systems but are very expensive and hence applicable mainly for very large machines.

(iii) Rotor Power Control: Speed/torque characteristics of the induction machine can be changed to suit for variable speed drive applications by controlling rotor power, but it is limited to wound rotor motor only. Both sub-synchronous and super-synchronous speed operations are possible by extracting or injecting electrical power in the rotor circuit. A large portion of slip power can be extracted out and used to drive some other electrical equipment, or fed back to the a.c. mains. In this regard Kramer and Scherbius principles are of great importance.

(iv) Rotor Impedence Control: The torque developed by the induction motor at a given speed can be varied by adjusting the effective rotor impedence (in case of slip ring motor). Previously, external passive elements like resistors, saturable reactors and capacitors were used to have high starting torque and wide range of speed variations. Recently SCRs are introduced in rotor circuit to control the effective rotor impedence and hence speed of the machine. Both phase controlled and chopper controlled circuits are used for the purpose by varying firing angle and ON-OFF time respectively.

1.3 LITERATURE REVIEW

1.3.1 Modelling and Simulation of Symmetrical Induction Machinary

Krause [5] has described a rigorous method of analysing the constant speed performance of symmetrical induction machine with applied stator phase voltage of any periodic form using two axes models, commonly used in digital simulation. This is sometimes referred to as the "Blondel two reaction method" [6]. It has also been shown that by employing a series of reference frames, d.c. circuit theory may be used to determine the steady state performance for non-sinusoidal operation which had been limited to analysis by the method of symmetrical components earlier. One of the major advantages of using orthogonal axes models is that the time dependence of the inductance coefficients is eliminated, equivalent stator to rotor mutual inductance becomes constant by describing the rotor variables in stationary reference frame.

Krause and Thomas [7] have described the simulation technique which would be useful in studying the performance of induction machine when used in conjunction with electronic switches in the stator. Also examples depicted in the above-mentioned paper serves us as guideposts in selecting the best reference frame or the best combination of reference frames in multimachine studies.

A generalized system model for all types of slip ring machinesincluding synchronous and induction motor is presented by Ramshaw and Padiyar [8].

The representation of induction motor at constant speed in terms of variable inductance coefficients and instantaneous winding currents has been reported by Shepherd [9]. However, no general solution is obtained and even developed torque is a function of inductance coefficients which are time variant in the steady state. So, as compared with the two axis model [5, 7] this technique is not acceptable as computation is more complex.

1.3.2 Analysis of Induction Motor Drives with Thyristor Inverters

Thyristor inverter generates a periodic non-sinusoidal voltage for which state variable method and fast Fourier transformation method have been proposed to date for performance analysis of induction motor. Probably, the most popular approaches for the analysis of a.c. motor drives which employ non-sinusoidal sources are harmonic superposition technique using symmetrical components [7] or method of multiple reference frame [5]. In the examples considered by Jacovides [10], the voltage applied to the motor is assumed to be rectangular, however in practical systems the applied voltage is not rectangular and method should not be used for predicting current waveforms. Besides the motor losses using this method can be quite significant if motors having extremely low p.u. rated slip are used.

The experimental thyristor power control circuits shown by Paice [11] are practically interesting where importance of input currents in simple induction motor speed

control systems are discussed and it is proved that six thyristor power circuit conjunction with star connected motor was found to be the least demanding.

Koga [12] demonstrated that instantaneous torque can be expressed in the form of infinite series containing function of induction machine constants. This makes it possible to grasp the overall characteristics of instantaneous torque.

A method of predicting the sixth harmonic electromagnetic torque of an induction motor arising from a rectifier inverter power source is presented by Lipo, Krause and Jordan [13]. This study includes both the effects of harmonic variation in rotor speed and inverter voltage. But effect of machine parameters on the ripple torque has not been mentioned and twelfth and higher harmonics are not considered though higher harmonics do not contribute appreciably to speed oscillation. For practical systems, this pulsation at low frequencies become greater than that predicted by a classical constant speed, constant voltage analysis.

The method for calculating the harmonic losses including separation in various components is presented by Klingshirn and Jordan [14] where it is shown that harmonic losses are to be nearly independent of motor load.

1.3.3 Pulse Width Modulated Inverters for a.c. Machine

It is possible to control the voltage harmonics of an induction motor fed by voltage source inverter either by modulating the voltage waveform within the inverter with the

aid of control logic for firing schemes or by using more than one inverter. Modulation within the inverter is referred to as pulse width modulation in literature [15, 16, 17]. The use of P.W.M. techniques applied to thyristor inverters to obtain variable frequency-variable voltage waveforms is reviewed by Mokrytzki [17]. The prime objective of such systems is to preserve a near sine wave envelope rather than to eliminate specific harmonics. This allows a potentially wider range of speed control by keeping harmonics in fixed but small proportions relative to fundamental frequency. However, before the application of such drives becomes feasible, the problems of complex control circuitry and relatively high motor losses must be solved.

Pollack [15] has discussed various P.W.M. techniques along with significant practical benefits on P.W.M. inverters. Normally comparison between different modulation techniques are based primarily on harmonic content in the output waveform.

Adams [16] has shown that adaptive ratio pulse width modulation scheme which combines the beneficial features of several available techniques and makes the necessary compromises to optimize the operating characteristics of a modulated six step inverter.

Generalized theoretical problem of eliminating up to five harmonics in half and full bridge inverter output waveforms was discussed by Patel and Hoft [18]. The feasibility of obtaining practically sinusoidal output voltage waveforms which are highly desirable in most inverter

applications is discussed. But implementation of the solutions is an involved problem. So results presented by Adams, Patel and Hoft [16, 18] should provide techniques that are practical and economical, considering the availability of low cost and sophisticated integrated circuit components for inverter control.

1.4 SUMMARY OF THE WORK

In Chapter 2, the D/Q model of the induction motor, with axes attached to the stator has been used throughout the analysis. A brief description of D/Q axes model for three phase induction motor is given in section 2.2. Afterwards, machine equations are used to derive the per phase equivalent circuit for sinusoidal excitation. An ideal three phase full bridge voltage source inverter has also been considered in this section to determine the different voltage levels in the stator phases of a star connected (3 wire) squirrel cage induction motor for fundamental complete cycle, neglecting circuit commutation. Lastly, primitive machine equations are used to find out all the stator phase voltages.

In Chapter 3, steady state analysis for motor current in frequency domain has been carried out. Motor equivalent circuit is used to find out the relation between stator and rotor currents as well as to determine the equivalent impedence of the said circuit for any harmonic frequency. Each and every variable which decide the status

of stator and rotor current waveforms are considered separately. Behaviour of existing predominant harmonics present in the line current with the change in inverter operating frequency and machine parameters has also been discussed in detail.

In Chapter 4, the expression of electromagnetic torque from D/Q model is derived for stationary reference frame attached to the stator. Nature of the above-mentioned torque in V.S.I. fed induction motor is discussed. With the help of the fundamental expression for instantaneous electromagnetic torque, the expression for the aforesaid torque in six step V.S.I. fed induction motor is also carried out to find the steady and pulsating harmonic torques. The speed versus average torque and speed versus sixth harmonic torque characteristics are computed, using computer programme. Instantaneous waveform for pulsating harmonic torque have been plotted using computer. A study on the effect of the changes in rotor time constant and inverter operating frequency on the above-mentioned characteristics has been carried out using simulated waveforms.

In Chapter 5, a detailed study of the effect of modulated inverter output voltage on current and torque harmonics has been carried out. Basic operation of an ideal three phase pulse width modulated inverter with 180° conduction control as well as pulse width modulation

techniques are discussed. Afterwards, characteristics of equal pulse width waveforms are also taken care off. Stress has been given to all predominant existing harmonics of the said waveform to find out the nature of stator current and electromagnetic torque. Effect of modulation index and number of pulses per half cycle on current and torque harmonics have been discussed in detail in this chapter. However, by a certain choice of the parameter of the modulating waveforms, it is possible to reduce certain torque harmonies.

Chapter - 2

D/Q MODEL OF INDUCTION MOTOR IN STATIONARY REFERENCE FRAME

2.1 INTRODUCTION

To develop a general method of analysis of rotating machine the underlying method depicted in the next section is very useful. Simulation of any problem on electric machine can be done using this method. The dynamic characteristics of induction motor influence the overall performance of the system of which it is a part in many applications. In order to use digital computer effectively it is important to have an induction machine representation which can easily be arranged to simulate many modes of operation. To determine the performance of induction machinery with any periodic voltage or current applied to the stator, the method of analysing using a series of reference frames can be used with the help of d.c. circuit theory. The development of this method of multiple reference frame has been done by Krause [5]. It involves the derivation of operating equations of the machine by a two axes oriented D/Q transformation from those primitive machine equations. This relation can be applied to a variety of important specific problems of the rotating machine.

In Section 2.2, the derivation of differential equations describing the transient response behaviour of induction machine and transformation from three phase balance system to D/Q frame attached to stator has been carried out.

Derivation of induction machine equivalent circuit from machine equation obtained in Section 2.2 has been done for sinusoidal excitation in Section 2.3.

In Section 2.4 an ideal full bridge voltage source inverter has been considered for finding the instantaneous voltage level in the stator phases of a star connected (3 wire) three phase squirrel cage induction motor. Depending upon the conduction period of thyristors, i.e. 120° or 180°, one fundamental complete cycle has been considered neglecting circuit commutation.

2.2 INDUCTION MACHINE EQUATIONS IN STATIONARY REFERENCE FRAME

In this section the equations of motor performance and transformation from three phase balanced system to D/Q frame have been outlined. To facilitate D/Q analysis, the following assumptions have been made:

(i) uniform air gap between stator and rotor,

(ii) linear magnetic circuit, saturation is ignored so superposition can be employed.

(iii) identical stator windings distributed so as to produce a sinusoidal m.m.f. wave in space, with the phases and arranged so that only one rotating m.m.f. wave is established by a balanced three phase stator currents.

(iv) rotor coils or bars arranged so that at any fixed time, the rotor m.m.f. wave can be considered to

be a space sinusoid having the same number of poles as the stator m.m.f. wave.

(v) leakage inductances of the phases of the machine are considered to be independent and lumped, further it is assumed that stator and rotor leakage inductances are constant at all frequencies.

So machine is regarded as a group of linear coupled circuits.

The symmetrical three phase induction machine shown in Figure 2.1 will be employed for development of equations. With appropriate subscript, i.e. a, b, c (for stator quantities) and A, B, C (for rotor quantities), the following voltage equation is applicable to each phase winding of stator and rotor.

$$v = p\lambda + r.i \tag{2.1}$$

where

λ : total flux linkage per phase

r : winding resistance per phase

p : the operator d/dt

v and i are phase voltage and phase current respectively.

The flux linkage equations for a three wire machine may be expressed as,

$$\lambda_a = L_{aa}.i_a + L_{ab}(i_b + i_c) + L_{aA}[i_A.\mathrm{Cos}\theta_2 + i_B.\mathrm{Cos}(\theta_2 + 120°) + i_C.\mathrm{Cos}(\theta_2 - 120°)] \tag{2.2a}$$

$$\lambda_b = L_{aa} \cdot i_b + L_{ab}(i_c + i_a) + L_{aA}[i_A \cdot \mathrm{Cos}(\theta_2 - 120°)$$
$$+ i_B \cdot \mathrm{Cos}\theta_2 + i_C \cdot \mathrm{Cos}(\theta_2 + 120°)] \tag{2.2b}$$

$$\lambda_c = L_{aa} \cdot i_c + L_{ab}(i_a + i_b) + L_{aA}[i_A \cdot \mathrm{Cos}(\theta_2 + 120°)$$
$$+ i_B \cdot \mathrm{Cos}(\theta_2 - 120°) + i_C \cdot \mathrm{Cos}\theta_2] \tag{2.2c}$$

\- in stator side

and

$$\lambda_A = L_{AA} \cdot i_A + L_{AB}(i_B + i_C) + L_{aA}[i_a \cdot \mathrm{Cos}\theta_2$$
$$+ i_b \cdot \mathrm{Cos}(\theta_2 - 120°) + i_c \cdot \mathrm{Cos}(\theta_2 + 120°)] \tag{2.3a}$$

$$\lambda_B = L_{AA} \cdot i_B + L_{AB}(i_C + i_A) + L_{aA}[i_a \cdot \mathrm{Cos}(\theta_2 + 120°)$$
$$+ i_b \cdot \mathrm{Cos}\theta_2 + i_c \cdot \mathrm{Cos}(\theta_2 - 120°)] \tag{2.3b}$$

$$\lambda_C = L_{AA} \cdot i_C + L_{AB}(i_A + i_B) + L_{aA}[i_a \cdot \mathrm{Cos}(\theta_2 - 120°)$$
$$+ i_b \cdot \mathrm{Cos}(\theta_2 + 120°) + i_c \cdot \mathrm{Cos}\theta_2] \tag{2.3c}$$

\- in rotor side

where

L_{aa}, L_{AA} : self inductances of stator and rotor winding respectively,

L_{ab}, L_{AB} : mutual inductances between phases of stator and rotor winding respectively,

L_{aA} : peak value of stator to rotor mutual inductance,

θ_2 : angular displacement between stator and rotor axis.

[as $L_{aa} = L_{bb} = L_{cc}$ and $L_{ab} = L_{bc} = L_{ca}$ in stator side

$L_{AA} = L_{BB} = L_{CC}$ and $L_{AB} = L_{BC} = L_{CA}$ in rotor side

and $L_{aA} = L_{bB} = L_{cC}$].

As mutual inductance vary with the displacement angle θ_2, so time varying coefficients will appear in the voltage equations. This undesirable feature can be eliminated by proper change of variables, which leads to transformation of voltages and currents of both stator and rotor to a common frame of reference. It is helpful, however, to correlate the change of variables to trigonometric relationship which exist between sets of axes. Figure 2.1 shows the angular relation of stator and rotor axis of a three phase machine with the new set which is an orthogonal set (D/Q axis) rotating at an arbitrary electrical angular velocity 'ω_e'. So it is clear that a-b-c set is fixed in the stator and A-B-C set is fixed in rotor and hence rotates at an angular velocity $d\theta_2/dt$. The angular relationship between three sets of axis at time origin (t = 0) can be selected arbitrarily.

The transformation equation which can be correlated to relative angular positions of the axes in the Figure 2.1 are written as follows:

With all three stator phases excited component m.m.fs along D/Q axes are

$$f_{1q} = N_a[f_a.\text{Cos}\ \omega_e t + f_b.\text{Cos}(\omega_e t - 120°) + f_c.\text{Cos}(\omega_e t + 120°)] \quad (2.4a)$$

$$f_{1d} = N_a[f_a . \text{Sin}\ \omega_e t + f_b . \text{Sin}(\omega_e t - 120°) + f_c . \text{Sin}(\omega_e t + 120°)]$$

- in stator side (2.4b)

and

$$f_{2q} = N_A[f_A . \text{Cos}\ \Theta_s + f_B . \text{Cos}(\Theta_s - 120°) + f_C . \text{Cos}(\Theta_s + 120°)] \quad (2.5a)$$

$$f_{2d} = [N_A\ f_A . \text{Sin}\ \Theta_s + f_B . \text{Sin}(\Theta_s - 120°) + f_C . \text{Sin}(\Theta_s + 120°)]$$

- in rotor side (2.5b)

where

N_a : effective turns/stator phase

N_A : effective turns/rotor phase

Θ_s : angular displacement between rotor phase A and the Q axis.

Now if

$$f_{1d} = \frac{N_a \cdot i_{1d}}{K_d} \quad \text{and} \quad f_{1q} = \frac{N_a \cdot i_{1q}}{K_q}$$

where K_d and K_q are arbitrary constants, and if K_d and K_q are taken as 2/3 in order to simplify the numerical co-efficient in the later equations, then

$$i_{1q} = \frac{2}{3}\ [i_a . \text{Cos}\ \omega_e t + i_b . \text{Cos}(\omega_e t - 120°) + i_c . \text{Cos}(\omega_e t + 120°)] \quad (2.6a)$$

$$i_{1d} = \frac{2}{3}\ [i_a . \text{Sin}\ \omega_e t + i_b . \text{Sin}(\omega_e t - 120°) + i_c . \text{Sin}(\omega_e t + 120°)] \quad (2.6b)$$

Therefore, i_{1q}, i_{1d} will denote Q axis and D axis instantaneous stator currents respectively.

The reverse relation, obtained by simultaneous solution of equation(2.6a) and (2.6b) in conjunction with the equation $i_a + i_b + i_c = 0$ are:

$$i_a = i_{1q}.\mathrm{Cos}\,\omega_e t + i_{1d}.\mathrm{Sin}\,\omega_e t \qquad (2.7a)$$

$$i_b = i_{1q}.\mathrm{Cos}(\omega_e t - 120°) + i_{1d}.\mathrm{Sin}(\omega_e t - 120°) \qquad (2.7b)$$

$$i_c = i_{1q}.\mathrm{Cos}(\omega_e t + 120°) + i_{1d}.\mathrm{Sin}(\omega_e t + 120°) \qquad (2.7c)$$

Similarly for rotor currents also.

So in the equations (2.4) and (2.5) 'f' can represent either voltage, current or flux linkage and it is seen that transformation equations are valid regardless of the form of the voltage and current in either the stator or rotor. Again if, however, only balanced condition is to be considered, the three voltages or currents are defined by any two, i.e. a third substitute variable is unnecessary. Then,

$$\lambda_{1q} = \frac{2}{3}[\lambda_a.\mathrm{Cos}\,\omega_e t + \lambda_b.\mathrm{Cos}(\omega_e t - 120°) + \lambda_c.\mathrm{Cos}(\omega_e t + 120°)] \qquad (2.8a)$$

$$\lambda_{1d} = \frac{2}{3}[\lambda_a.\mathrm{Sin}\,\omega_e t + \lambda_b.\mathrm{Sin}(\omega_e t - 120°) + \lambda_c.\mathrm{Sin}(\omega_e t + 120°)] \qquad (2.8b)$$

Now combining equations (2.8a) and (2.8b) with equations (2.2a), (2.2b) and (2.2c),

$$\lambda_{1q} = L_{11}.i_{1q} + \frac{3}{2} L_{aA}.i_{2q} \qquad (2.9a)$$

and $$\lambda_{1d} = L_{11}.i_{1d} + \frac{3}{2} L_{aA}.i_{2d} \qquad (2.9b)$$

where $L_{11} = L_{aa} - L_{ab}$ and i_{2q}, i_{2d} will denote instantaneous Q axis and D axis rotor currents respectively.

Similarly for rotor side,

$$\lambda_{2q} = L_{22}.i_{2q} + \frac{3}{2} L_{aA}.i_{1q} \tag{2.10a}$$

and $$\lambda_{2d} = L_{22}.i_{2d} + \frac{3}{2} L_{aA}.i_{1d} \tag{2.10b}$$

where $L_{22} = L_{AA} - L_{AB}$.

Again expression for D/Q component of the stator voltages are as follows:

$$v_{1q} = \frac{2}{3}[v_a.\text{Cos}\ \omega_e t + v_b.\text{Cos}(\omega_e t - 120°) + v_c.\text{Cos}(\omega_e t + 120°)] \tag{2.11a}$$

and

$$v_{1d} = \frac{2}{3}[v_a.\text{Sin}\ \omega_e t + v_b.\text{Sin}(\omega_e t - 120°) + v_c.\text{Sin}(\omega_e t + 120°)] \tag{2.11b}$$

But for phase'a', $v_a = r_1.i_a + p\lambda_a$ (from equation 2.1)

$$i_a = i_{1q}.\text{Cos}\ \omega_e t + i_{1d}.\text{Sin}\ \omega_e t$$ (from equation 2.7a)

Similarly for stator flux linkage,

$$\lambda_a = \lambda_{1q}.\text{Cos}\ \omega_e t + \lambda_{1d}.\text{Sin}\ \omega_e t$$

i_b, λ_b and i_c, λ_c can also be expressed with the help of equations (2.7b) and (2.7c) respectively.

So after simplified,

$$v_{1q} = r_1.i_{1q} + p\ \lambda_{1q} + \omega_e . \lambda_{1d} \tag{2.12a}$$

$$v_{1d} = r_1.i_{1d} + p\ \lambda_{1d} - \omega_e . \lambda_{1q} \tag{2.12b}$$

Again similarly for rotor quantities,

$$v_{2q} = r_2 \cdot i_{2q} + p\,\lambda_{2q} + \lambda_{2d}(p\theta_s) \quad (2.13a)$$

$$v_{2d} = r_2 \cdot i_{2d} + p\,\lambda_{2d} - \lambda_{2q}(p\theta_s) \quad (2.13b)$$

where r_1 : stator resistance/phase

r_2 : rotor resistance/phase

v_{2q}, v_{2d} are Q axis and D axis rotor voltages respectively.

In general, the machine parameters are measured with respect to stator winding with rotor variables referred to stator windings. The self inductance is separated into leakage inductance component and magnetizing inductance component to obtain the following voltage equations,

$$v_{1q} = r_1 \cdot i_{1q} + p\,\lambda_{1q} + \omega_e \cdot \lambda_{1d} \quad (2.14)$$

$$v_{1d} = r_1 \cdot i_{1d} + p\,\lambda_{1d} - \omega_e \cdot \lambda_{1q} \quad (2.15)$$

$$v'_{2q} = r'_2 \cdot i'_{2q} + p\,\lambda'_{2q} + \lambda'_{2d}(p\theta_s) \quad (2.16)$$

$$v'_{2d} = r'_2 \cdot i'_{2d} + p\,\lambda'_{2d} - \lambda'_{2q}(p\theta_s) \quad (2.17)$$

where

$$\lambda_{1q} = L_{1s} \cdot i_{1q} + M(i_{1q} + i'_{2q})$$

$$\lambda_{1d} = L_{1s} \cdot i_{1d} + M(i_{1d} + i'_{2d})$$

$$\lambda'_{2q} = L'_{2s} \cdot i'_{2q} + M(i_{1q} + i'_{2q})$$

$$\lambda'_{2d} = L'_{2s} \cdot i'_{2d} + M(i_{1d} + i'_{2d})$$

$$M = \frac{3}{2} L_{aA}, \qquad L_{1s} = L_{11} - \frac{3}{2}\left(\frac{N_a}{N_A}\right) \cdot L_{aA}$$

$$L'_{2s} = L'_{22} - \frac{3}{2}\left(\frac{N_a}{N_A}\right) \cdot L_{aA}$$

and quantities with prime are rotor quantities referred to stator.

Now equations (2.14), (2.15), (2.16) and (2.17) may be expressed in matrix form as

$$\begin{bmatrix} v_{1q} \\ v_{1d} \\ v'_{2q} \\ v'_{2d} \end{bmatrix} = \begin{bmatrix} (r_1+L_{11}p) & \omega_e L_{11} & Mp & \omega_e M \\ -\omega_e L_{11} & (r_1+L_{11}p) & -\omega_e M & Mp \\ Mp & (\omega_e-\omega_r)M & (r'_2+L'_{22}p) & (\omega_e-\omega_r)L'_{22} \\ -(\omega_e-\omega_r)M & Mp & -(\omega_e-\omega_r)L'_{22} & (r'_2+L'_{22}p) \end{bmatrix} \times \begin{bmatrix} i_{1q} \\ i_{1d} \\ i'_{2q} \\ i'_{2d} \end{bmatrix} \tag{2.18}$$

It is clear that voltage equations of symmetrical induction machine may be expressed in any reference frame by setting the speed of arbitrary reference frame ω_e is equal to the speed of desired reference frame. For example, selecting a stationary reference frame we get from the transformation equations (2.6) and (2.7) by substituting $\omega_e = 0$

$$\begin{bmatrix} i_{1q} \\ i_{1d} \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & -3/2 & 3/2 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{2.19}$$

and

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/2 & -3/2 \\ -1/2 & 3/2 \end{bmatrix} \begin{bmatrix} i_{1q} \\ i_{1d} \end{bmatrix} \tag{2.20}$$

And, voltage equations expressed in stationary reference frame may be obtained by setting $\omega_e = 0$ in the equation (2.18) for squirrel cage induction motor (i.e. $v'_{2q} = v'_{2d} = 0$), so equation (2.18) reduces to the following form

$$\begin{bmatrix} v_{1q} \\ v_{1d} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} (r_1+L_{11}p) & 0 & Mp & 0 \\ 0 & (r_1+L_{11}p) & 0 & Mp \\ Mp & -\omega_r M & (r'_2+L'_{22}p) & -\omega_r L'_{22} \\ \omega_r M & Mp & \omega_r L'_{22} & (r'_2+L'_{22}p) \end{bmatrix} \times \begin{bmatrix} i_{1q} \\ i_{1d} \\ i'_{2q} \\ i'_{2d} \end{bmatrix} \qquad (2.21)$$

This equation is used for the analysis of voltage source inverter fed induction motor from this section onwards.

2.3 DERIVATION OF INDUCTION MACHINE EQUIVALENT CIRCUIT FROM MACHINE EQUATION FOR SINUSOIDAL EXCITATION

Consider that the motor is operating at constant speed with balanced voltages

$$v_a = \sqrt{2}\, V_1.\mathrm{Cos}\,\omega_s t$$
$$v_b = \sqrt{2}\, V_1.\mathrm{Cos}(\omega_s t - 120°)$$
$$v_c = \sqrt{2}\, V_1.\mathrm{Cos}(\omega_s t + 120°)$$

applied to the stator with rotor shorted. Let the three stator and referred rotor currents are

$$i_a = \sqrt{2}\, I_1.\text{Cos}(\omega_s t + \alpha_1)$$

$$i_b = \sqrt{2}\, I_1.\text{Cos}(\omega_s t + \alpha_1 - 120°)$$

$$i_c = \sqrt{2}\, I_1.\text{Cos}(\omega_s t + \alpha_1 + 120°)$$

and
$$i_A = \sqrt{2}\, I_2'.\text{Cos}(\omega_s t + \alpha_2)$$

$$i_B = \sqrt{2}\, I_2'.\text{Cos}(\omega_s t + \alpha_2 - 120°)$$

$$i_C = \sqrt{2}\, I_2'.\text{Cos}(\omega_s t + \alpha_2 + 120°)$$

where V_1, I_1 and I_2' are the r.m.s. values of stator phase voltage, stator and referred rotor line currents respectively. α_1, α_2 are the arbitrary phase difference angles with the stator voltage. With the help of matrix equation (2.19) for D/Q transformation under stationary reference frame, the component quantities for stator voltage, stator line current and referred rotor line current are as follows:

Stator voltages:
$$v_{1q} = \sqrt{2}\, V_1.\text{Cos}\ \omega_s t\ ,$$

$$v_{1d} = -\sqrt{2}\, V_1.\text{Sin}\ \omega_s t$$

$$= \sqrt{2}\, V_1.\text{Cos}(\omega_s t + \pi/2)$$

Stator currents:
$$i_{1q} = \sqrt{2}\, I_1.\text{Cos}(\omega_s t + \alpha_1)$$

$$i_{1d} = -\sqrt{2}\, I_1.\text{Sin}(\omega_s t + \alpha_1)$$

$$= \sqrt{2}\, I_1.\text{Cos}(\omega_s t + \alpha_1 + \pi/2)$$

Rotor currents:
$$i_{2q}' = \sqrt{2}\, I_2'.\text{Cos}(\omega_s t + \alpha_2)$$

$$i_{2d}' = -\sqrt{2}\, I_2'.\text{Sin}(\omega_s t + \alpha_2)$$

$$= \sqrt{2}\, I_2'.\text{Cos}(\omega_s t + \alpha_2 + \pi/2)$$

This is naturally to be expected on sinusoidal balanced voltages injected in the three phases. In steady state analysis we are usually interested in relations involving r.m.s. currents and voltages rather than instantaneous values. A convenient link between D/Q variables and r.m.s. phase variables was already furnished by matrix equation (2.20).

As stated earlier, the Q axis is so placed that it coincides with the axis of phase 'a' at t = 0, so now Q axis quantities can be written in phasor form as follows,

$\bar{i}_{1q} = \sqrt{2}\, I_1 \angle \alpha_1$; $\bar{i}'_{2q} = \sqrt{2}\, I'_2 \angle \alpha_2$; $\bar{v}_{1q} = \sqrt{2}\, V_1 \angle 0°$

again

$\bar{i}_{1d} = \sqrt{2}\, I_1 \angle(\alpha_1 + \frac{\pi}{2}) = j\, i_{1q}$; $\bar{i}'_{2d} = \sqrt{2}\, I'_2 \angle(\alpha_2 + \frac{\pi}{2}) = j\, i'_{2q}$

and $\bar{v}_{1d} = \sqrt{2}\, V_1 \angle \frac{\pi}{2} = j\, v_{1q}$.

From machine equation (2.21) we can take out first and fourth sub-equations for finding out the machine equivalent circuit.

Considering fourth sub-equation of the matrix equation (2.21)

$$0 = \omega_r M . i_{1q} + Mp . i_{1d} + \omega_r L'_{22} . i'_{2q} + (r'_2 + L'_{22} p) i'_{2d}$$

substituting $p = j\omega_s$ (for sinusoidal input) we can get the steady state machine equation involving the relation between stator and rotor currents as follows:

$$0 = \omega_r M . \bar{i}_{1q} + j\omega_s M . \bar{i}_{1d} + \omega_r L'_{22} . \bar{i}'_{2q} + (r'_2 + j\omega_s L'_{22}) \bar{i}'_{2d}$$

or $$0 = j\omega_s M . \bar{i}_{1q} + (\frac{r'_2}{s} + j\omega_s L'_{22}) \bar{i}'_{2q}$$

where slip $S = (\frac{\omega_s - \omega_r}{\omega_s})$

Now substituting for $\bar{I}_{1q}$ and $\bar{I}'_{2q}$

$$j\omega_s M.I_1 + (\frac{r'_2}{s} + j\omega_s L'_{22})I'_2 = 0 \qquad (2.22)$$

Again to find the relation between stator voltage and motor current at steady state we have the first sub-equation from the matrix equation (2.21)

$$v_{1q} = (r_1 + L_{11}p)i_{1q} + Mp\, i'_{2q}$$

or

$$\bar{v}_{1q} = (r_1 + j\omega_s L_{11})\bar{I}_{1q} + j\omega_s M.\bar{I}'_{2q}$$

(substituting $p = j\omega_s$)

Now substituting for $\bar{v}_{1q}$, $\bar{I}_{1q}$ and $\bar{I}'_{2q}$ we get

$$V_1 = (r_1 + j\omega_s L_{11})I_1 + j\omega_s M.I'_2 \qquad (2.23)$$

So combining equations (2.22) and (2.23) in matrix form

$$\begin{bmatrix} V_1 \\ 0 \end{bmatrix} = \begin{bmatrix} (r_1+j\omega_s L_{11}) & j\omega_s M \\ j\omega_s M & (\frac{r'_2}{s} + j\omega_s L_{22}) \end{bmatrix} \times \begin{bmatrix} I_1 \\ I'_2 \end{bmatrix} \qquad (2.24)$$

Equation (2.24) can also be written as

$$\begin{bmatrix} V_1 \\ 0 \end{bmatrix} = \begin{bmatrix} r_1+j\omega_s(L_{11}-M)+j\omega_s M & j\omega_s M \\ j\omega_s M & \frac{r'_2}{s} + j\omega_s(L'_{22}-M)+j\omega_s M \end{bmatrix} \times \begin{bmatrix} I_1 \\ I'_2 \end{bmatrix}$$

(2.25)

This equation (2.25) suggests the equivalent circuit shown in Figure 2.2(a).

Again the following matrix equation is also valid for the circuit shown in Figure 2.2(b) where in place of I_2' we have considered I_2, where $I_2' = I_2 \left(\frac{N_A}{N_a}\right)$

$$\begin{bmatrix} V_1 \\ 0 \end{bmatrix} = \begin{bmatrix} r_1 + j\omega_s(L_{11}-M) + j\omega_s M & j\omega_s M\left(\frac{N_a}{N_A}\right) \\ j\omega_s M\left(\frac{N_a}{N_A}\right) & \left(\frac{N_a}{N_A}\right)^2 \left\{ \frac{r_2}{s} + j\omega_s(L_{22}-M) + j\omega_s M \right\} \end{bmatrix} \times \begin{bmatrix} I_1 \\ I_2\left(\frac{N_A}{N_a}\right) \end{bmatrix} \qquad (2.26)$$

But commonly the reference direction of rotor current is opposite to that of I_2'. So equation (2.26) can be written as

$$\begin{bmatrix} V_1 \\ 0 \end{bmatrix} = \begin{bmatrix} (r_1 + j\omega_s L_{11}) & j\omega_s M_m \\ j\omega_s M_m & \left(\frac{r_2'}{s} + j\omega_s L_{22}'\right) \end{bmatrix} \times \begin{bmatrix} I_1 \\ -I_r' \end{bmatrix} \qquad (2.27)$$

and corresponding equivalent circuit is shown in Figure 2.2(c).

where leakage inductance of stator $= (L_{11} - M)$

leakage inductance of rotor $= (L_{22}' - M_m)$

and magnetising inductance $M_m = M\left(\frac{N_a}{N_A}\right)$ and $r_2' = r_2\left(\frac{N_a}{N_A}\right)^2$.

2.4 DISTRIBUTION OF INPUT D.C. VOLTAGE ACROSS THE STATOR PHASE IN AN IDEAL VOLTAGE SOURCE INVERTER

Figure 2.3 shows the voltage source bridge inverter feeding three phase star connected load. The constant voltage source is obtained from a three phase voltage converter along with L-C filter. Here two types of inverter control technique are discussed:

(a) Three thyristor conduction or 180° conduction control technique, and

(b) Two thyristor conduction or 120° conduction control technique.

2.4.1 180° Conduction Control Technique

In this control three thyristors conduct at any instant as per the mode sequences. Either two thyristors from top half and one from the bottom half or two thyristors from bottom half and one from the top half conduct. Each thyristor conducts for half the inverter time period.

Generation of basic six step waveform

The inverter power circuit shown in Figure 2.3 consists of six switching elements which are located between d.c. bus and the load. An analysis can be made of this power module by replacing each thyristor with a mechanical switch. Then the development of an a.c. waveform is accomplished by simply letting these switches conduct alternately over a given interval. The top switches (S_1, S_3 and S_5) creating the positive outputs and the bottom switches (S_4, S_6 and S_2) the negative outputs. The voltages swings are with respect to a theoretical d.c. neutral. This latter point was devised only to simplify the discussion. Figure 2.4 illustrates the waveforms derived from this alternate switching action. The indicated switches are sequenced to give a 120° phase displacement between the three phases. The instantaneous values

of v_{ao}, v_{bo} and v_{co} can be used to determine the actual output voltage waveforms shown in Figure 2.4(a).

Line to line voltages relationships will be as follows: $v_{ab} = v_{ao} - v_{bo}$, $v_{bc} = v_{bo} - v_{co}$ and $v_{ca} = v_{co} - v_{ao}$. The rêsultant waveforms v_{ab}, v_{bc} and v_{ca} are shown in Figure 2.4(b). The line to neutral waveform clearly indicating the six step envelope can be calculated and plotted with the aid of v_{ao}, v_{bo} and v_{co}.

$$v_{an} = \frac{2}{3} v_{ao} - \frac{1}{3}(v_{bo} + v_{co})$$

$$v_{bn} = \frac{2}{3} v_{bo} - \frac{1}{3}(v_{co} + v_{ao})$$

$$v_{cn} = \frac{2}{3} v_{co} - \frac{1}{3}(v_{ao} + v_{bo})$$

Such a representative wave is shown in Figure 2.4(c). Thus a basic six step waveform is achieved by the simple switching action of the three phase inverter power bridge.

This basic six step waveform can also be achieved for a star connected induction motor load, which is as follows.

From fundamental voltage current relationship for a winding ($v = r.i + p\lambda$), we get

$$v_{an} = r_1.i_a + p\lambda_a \qquad (2.28a)$$

$$v_{bn} = r_1.i_b + p\lambda_b \qquad (2.28b)$$

$$v_{cn} = r_1.i_c + p\lambda_c \qquad (2.28c)$$

where λ is the flux linkage with the winding denoted by subscripts. Voltage symbols denote voltages applied to the winding and currents are directed into the winding.

Depending upon the switching of the thyristors (Figure 2.3) for each mode (one sixth of the total time period) the following equations are valid:

Mode I $\quad v_{an} - v_{bn} = V_{dc}$; $\quad v_{cn} = v_{an}$ (2.29a)

Mode II $\quad v_{an} - v_{bn} = V_{dc}$; $\quad v_{bn} = v_{cn}$ (2.29b)

Mode III $\quad v_{bn} - v_{cn} = V_{dc}$; $\quad v_{an} = v_{bn}$ (2.29c)

Mode IV $\quad v_{bn} - v_{cn} = V_{dc}$; $\quad v_{cn} = v_{an}$ (2.29d)

Mode V $\quad v_{cn} - v_{an} = V_{dc}$; $\quad v_{bn} = v_{cn}$ (2.29e)

Mode VI $\quad v_{cn} - v_{an} = V_{dc}$; $\quad v_{an} = v_{bn}$ (2.29f)

Now adding equations (2.28a), (2.28b) and (2.28c)

$$v_{an} + v_{bn} + v_{cn} = r_1(i_a + i_b + i_c) + p(\lambda_a + \lambda_b + \lambda_c) \quad (2.30)$$

As $i_a + i_b + i_c = 0$ and $\lambda_a + \lambda_b + \lambda_c = 0$ at balanced condition, therefore, with the help of equations (2.29a) to (2.29f) for different modes, equation (2.30) becomes as follows:

Mode I $\quad 3v_{an} - V_{dc} = 0$ (2.31a)

Mode II $\quad 3v_{an} - 2V_{dc} = 0$ (2.31b)

Mode III $\quad 3v_{an} - V_{dc} = 0$ (2.31c)

Mode IV $\quad 3v_{an} + V_{dc} = 0$ (2.31d)

Mode V $\quad 3v_{an} + 2V_{dc} = 0$ (2.31e)

Mode VI $\quad 3v_{an} + v_{dc} = 0$ (2.31f)

So, v_{an} would be $V_{dc}/3$ (Mode I and III), $2V_{dc}/3$ (Mode II), $-V_{dc}/3$ (Mode IV and VI), and $-2V_{dc}/3$ (Mode V).

v_{bn} and v_{cn} can be known by substituting the values of v_{an} in the equations (2.29a) to (2.29f) for various modes.

Proceeding along similar line during modulation (Chapter 5), if top or bottom three SCRs are conducting then $v_{an} - v_{bn} = 0$, $v_{bn} - v_{cn} = 0$ and $v_{cn} - v_{an} = 0$

therefore, $$v_{an} = v_{bn} = v_{cn} \tag{2.32}$$

but as $i_a + i_b + i_c = 0$ and $\lambda_a + \lambda_b + \lambda_c = 0$ as well as $v_{an} + v_{bn} + v_{cn} = 0$ (equation 2.30), therefore

$$v_{an} = v_{bn} = v_{cn} = 0$$

2.4.2 120° Conduction Control Technique

In this control, two thyristors always conduct at a time as per the firing sequences, i.e. one thyristor from top and one from bottom half, and each thyristor conducts for one third of the total inverter time period. It means that one phase out of the three phases remains open all the time. It has been shown in Appendix 'B', that when the current through the remaining phase reduces to zero, from that instant only that particular phase will remain open circuited for a fraction of one sixth of the total time period. The time interval for which open circuitry occurs depends on the inverter frequency as well as the time constant of the load. Figure 2.5 shows the d.c. voltage distribution in all the three phases of the load for time period 'T' assumed load to be resistive. Typical voltage distribution as well as current waveform for R-L load have been shown in Appendix 'B'.

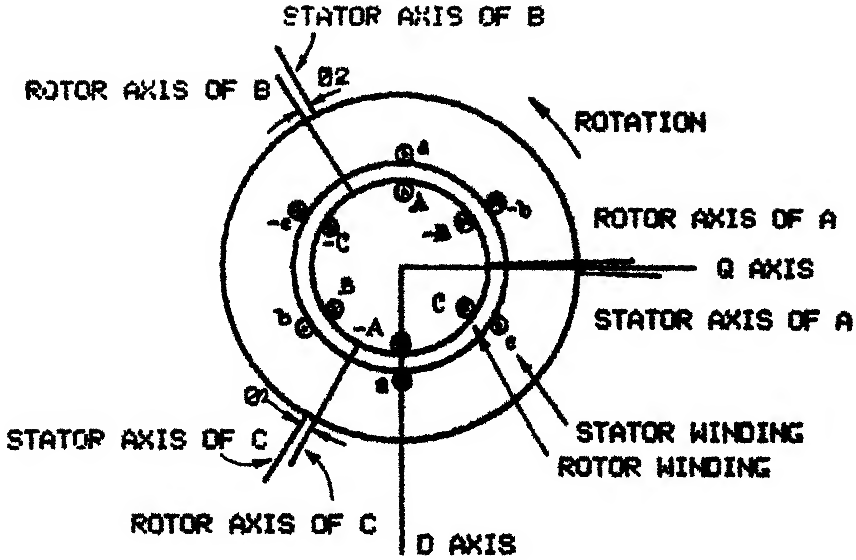


FIG 2 1(A) WE=0,T=0,ØS=0,Ø2 TENDS TO ZERO

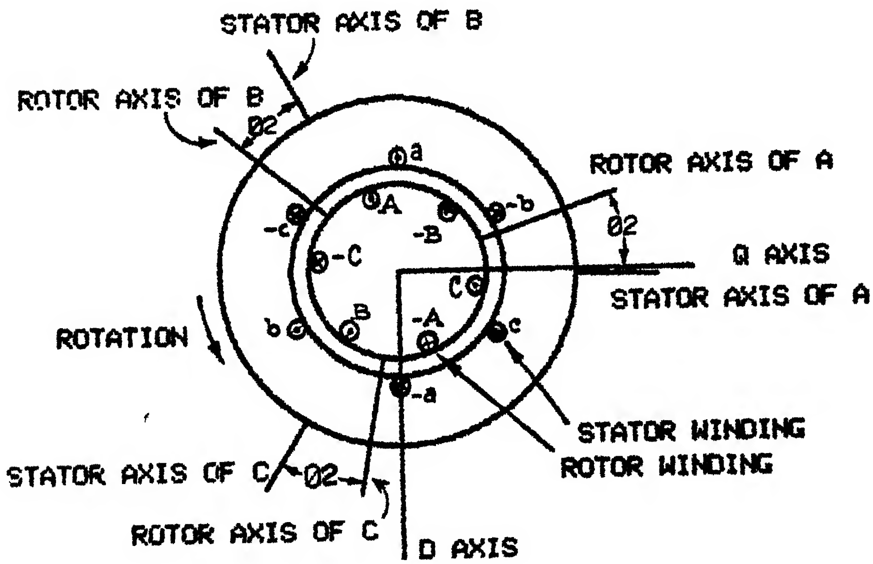


FIG 2 1(B) WE=0,T=T1,(Ø2=WR.T1)

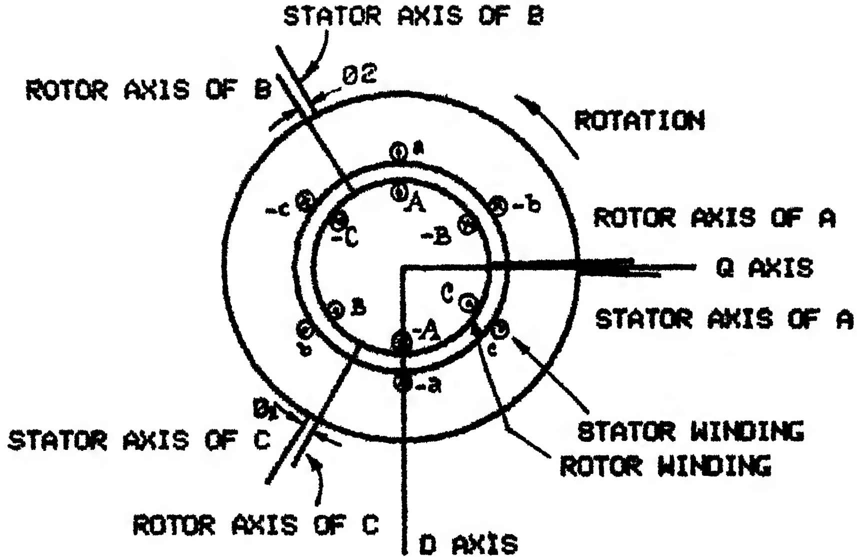


FIG 2 1(C) WE≠0,T=0, 02 TENDS TO ZERO

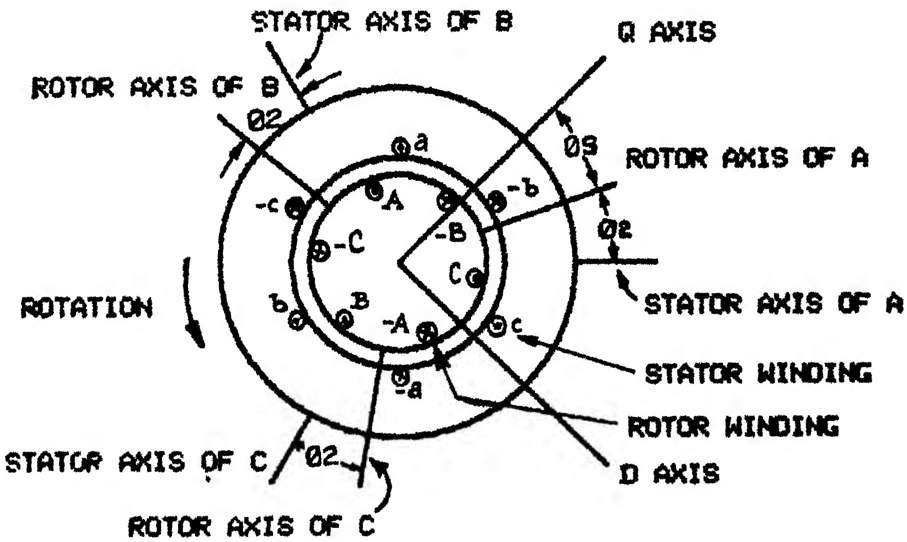


FIG 2 1(D) WE ≠ 0 , T=T1

FIG 2 1 STATOR & ROTOR MAGNETIC AXES OF THREE PHASE INDUCTION MACHINE ALONG WITH D/Q AXES

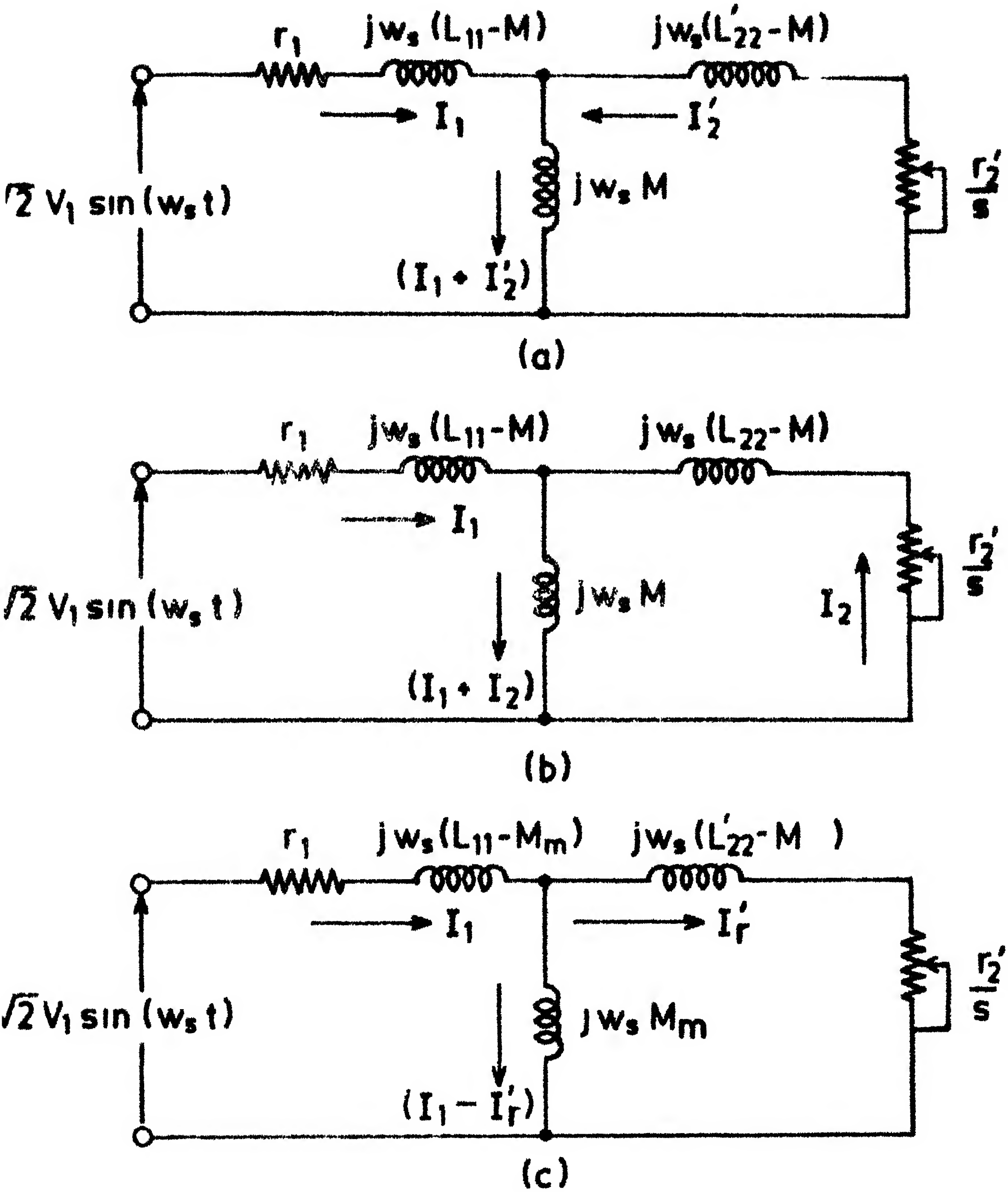


Fig 2.2 INDUCTION MOTOR EQUIVALENT CIRCUIT

(a) Corresponds to equation (2 25)

(b) Corresponds to equation (2 26)

(c) Corresponds to equation (2 27)

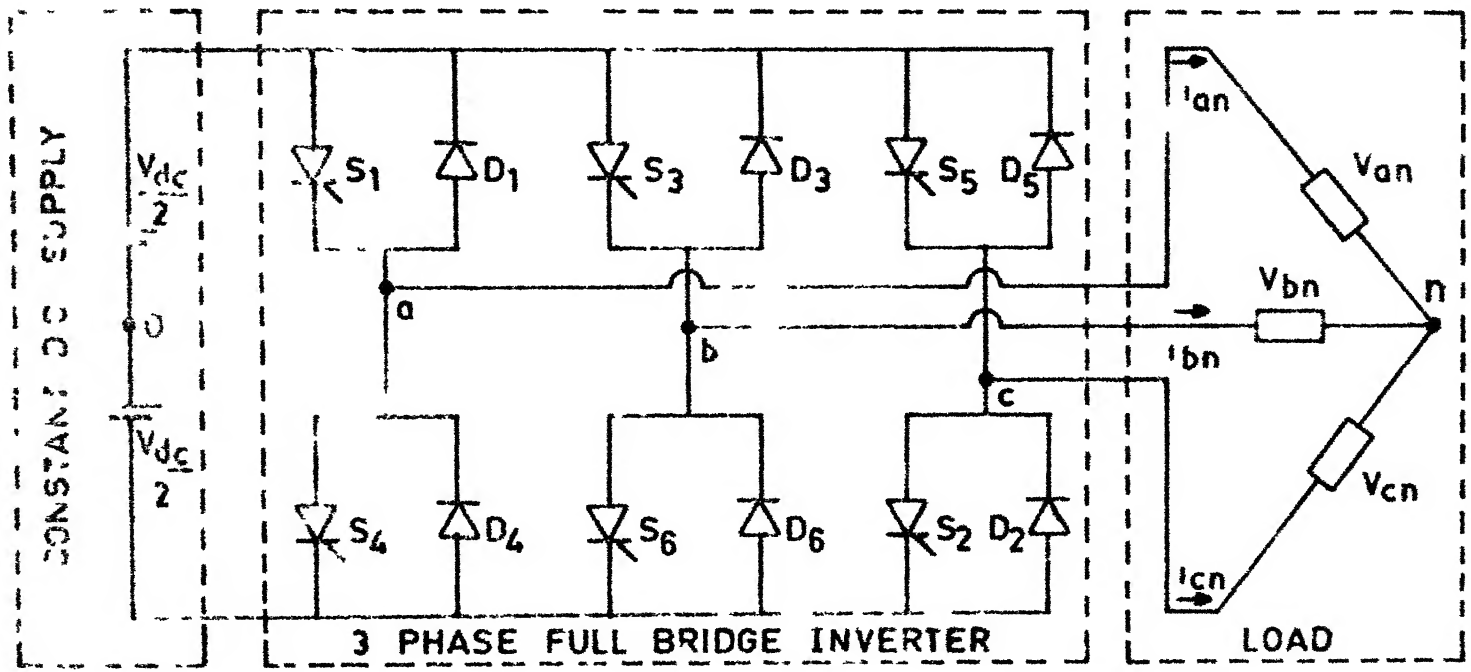
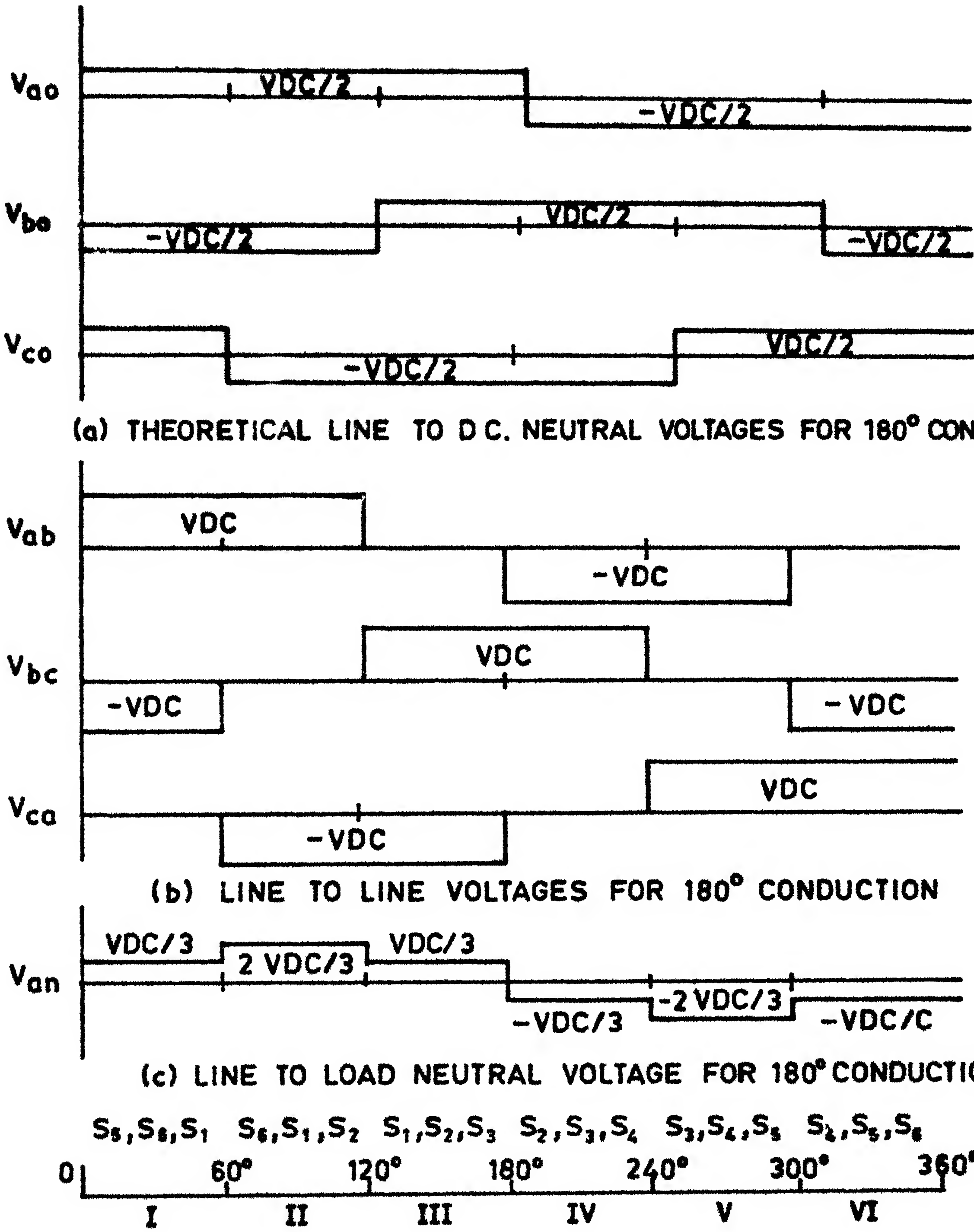


Fig 2.3 3 Ø INVERTER POWER CIRCUIT FEEDING STAR CONNECTED 3 WIRE LOAD



(a) THEORETICAL LINE TO D C. NEUTRAL VOLTAGES FOR 180° CONDUCTION

(b) LINE TO LINE VOLTAGES FOR 180° CONDUCTION

(c) LINE TO LOAD NEUTRAL VOLTAGE FOR 180° CONDUCTION

Fig. 2.4 VOLTAGE WAVEFORMS FOR 3 PHASE INVERTER POWER CIRCUIT.

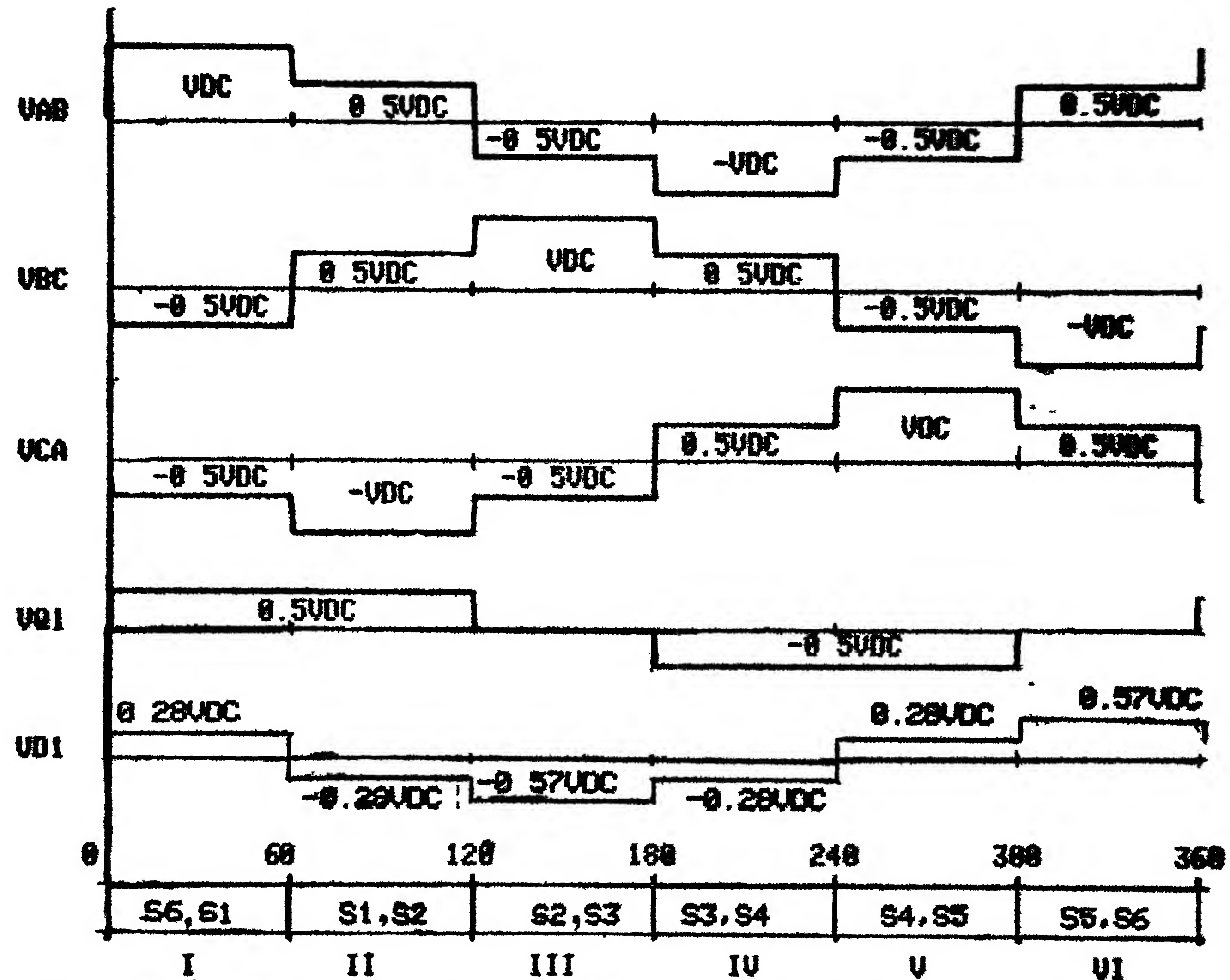


FIG 2 5 3 PHASE LINE & D/Q STATOR VOLTAGES FOR 120 DEGREE CONDUCTION

Chapter - 3

METHOD OF ANALYSIS IN FREQUENCY DOMAIN

3.1 INTRODUCTION

This chapter presents the steady state analysis for current of three phase squirrel cage induction motor fed by voltage source inverter having 180 degree conduction control technique. This analysis has been carried out for determining the motor current with all existing harmonics in inverter output voltage under different operating conditions. But in actual computation higher order harmonics have not been taken into consideration due to very low harmonic content.

Here motor equivalent circuit equations (2.22) and (2.23) of the section 2.3 have been considered to find out (i) the relation between stator and rotor currents and (ii) the effect of inverter output voltage on motor currents. These are primarily dependent on machine parameters, though operating condition is also important in this regard, but it is of secondary importance.

The important features which decide the status of stator current waveforms including all lower order harmonics are as follows:

(a) Fundamental stator r.m.s. current

(b) Harmonic stator r.m.s. current

(c) Fundamental power factor angle

(d) Harmonic power factor angle.

For rotor current waveforms including all predominant harmonics, four additional features need to be considered, viz.

(e) Magnitude relationship between stator fundamental and rotor fundamental currents

(f) Magnitude relationship between stator harmonic and rotor harmonic currents

(g) Phase relationship between stator fundamental and rotor fundamental currents

(h) Phase relationship between stator harmonic and rotor harmonic currents.

In section 3.2 Fourier analysis of the inverter output voltage has been carried out and it was transformed to D/Q axes for determining the motor currents in D/Q axes.

Frequency domain analysis has been done for stator voltage, stator and rotor currents in section 3.3. Firstly a relationship between stator and rotor currents was established in section 3.3.2 assuming possible expression for stator current which has been dealt in section 3.3.1. In section 3.3.3 rotor current variable was replaced by stator current variable. Fundamental and each existing harmonic of the stator voltage are defined in section 3.2, and each component of the above-mentioned voltage is also related with

corresponding stator current component of the motor line current for constant machine parameters (Appendix - 'A'). So fundamental and other harmonic components of the motor current (magnitude and phase angle difference with the corresponding voltage terms) can be determined.

Behaviour of these existing harmonics present in the line current for the machine under consideration with the change in inverter operating frequency and machine parameters has been discussed in section 3.4. In section 3.4.1 only ratio of r.m.s. harmonic current to fundamental r.m.s. current has been considered as a measure of individual harmonic content in stator current. Variations of fundamental power factor and harmonic power factor with motor speed have been depicted in section 3.4.2. These power factor angles play a very important role in determining the instantaneous current waveform.

In the last section of this chapter, effect of operating frequency and change of machine parameters on motor current for varying speed have been discussed. Section 3.5.1 and 3.5.2 are made separately for distinguishing the characteristics of fundamental and harmonic currents respectively. Effect of harmonic currents at different speeds of the induction motor on normal fundamental line current has been studied in section 3.5.3 for different inverter frequency operation.

3.2 EXPRESSION FOR HARMONIC CONTENT IN INPUT VOLTAGE

Only 180 degree conduction control technique of three phase bridge inverter shown in Figure 2.3 is considered here where three thyristors conduct at any instant. The voltage distribution of d.c. input voltage V_{dc} in all stator phases of the induction machine under all possible modes over the entire time period of T is known from Figure 2.4.

As motor terminal voltage is a periodic continuous function of time, then a Fourier series for the voltage can be obtained.

Stator phase 'a' to neutral voltage Figure 2.4(c)

$$v_{an} = v(\omega_s t) = V_{dc}/3 \text{ for } 0 < \omega_s t < \pi/3 \text{ and } 2\pi/3 < \omega_s t < \pi \quad (3.1a)$$

$$= -V_{dc}/3 \text{ for } \pi < \omega_s t < 4\pi/3 \text{ and } 5\pi/3 < \omega_s t < 2\pi \quad (3.1b)$$

$$= 2V_{dc}/3 \text{ for } \pi/3 < \omega_s t < 2\pi/3 \quad (3.1c)$$

$$= -2V_{dc}/3 \text{ for } 4\pi/3 < \omega_s t < 5\pi/3 \quad (3.1d)$$

where ω_s is the output angular frequency of the bridge inverter.

From Fourier method of wave form analysis,

$$v_{an} = v(\omega_s t) = \frac{2V_{dc}.\sin(\omega_s t)}{\pi} + \frac{2V_{dc}.\sin(5\omega_s t)}{5\pi} + \frac{2V_{dc}.\sin(7\omega_s t)}{7\pi} + \frac{2V_{dc}.\sin(11\omega_s t)}{11\pi} + \dots \infty$$

or, $v_{an} = v(\omega_s t) = \frac{2V_{dc}}{\pi} \Sigma \; \sin(n\omega_s t)/n$ (3.2a)

so, $v_{bn} = \frac{2V_{dc}}{\pi} \Sigma \; \sin n(\omega_s t - \frac{2\pi}{3})/n$ (3.2b)

and $v_{cn} = \frac{2V_{dc}}{\pi} \Sigma \; \sin n(\omega_s t + \frac{2\pi}{3})/n$ (3.2c)

where $n = 1,5,7,11,13,17,19,23,25\ldots\infty$.

After transformation in D/Q axes of equations (3.2a), (3.2b) and (3.2c)

$$v_{1q} = \frac{2V_{dc}}{\pi} \sum_{n=1,5,7,11}^{\infty} \sin(n\omega_s t)/n \qquad (3.3a)$$

and
$$v_{1d} = -\frac{2V_{dc}}{\pi}\left[\cos(\omega_s t) - \frac{\cos(5\omega_s t)}{5} + \frac{\cos(7\omega_s t)}{7} - \frac{\cos(11\omega_s t)}{11} + \ldots\ldots\ldots \infty\right]$$

or
$$v_{1d} = -\frac{2V_{dc}}{\pi}\left[\sum_{n=1,7,13}^{\infty} \frac{\cos(n\omega_s t)}{n} - \sum_{n=5,11,17}^{\infty} \frac{\cos(n\omega_s t)}{n}\right]$$

(3.3b)

Figure 3.1 shows the waveform of v_{1q} and v_{1d} as a periodic continuous function of time.

The stator voltage of the induction motor fed by V.S.I. is antisymmetric about $\omega_s t = \pi$, i.e. $v(\omega_s t) = -v(\omega_s t+\pi)$. This implies the absence of even harmonics in the input voltage fed to the motor. Again triplens are also absent because the sum of the voltages in all three phases is zero in this case of three phase three wire motor connection.

3.3 FREQUENCY DOMAIN EXPRESSIONS FOR STATOR VOLTAGE, STATOR AND ROTOR CURRENTS

In the last section it was noted that input voltage fed to the stator windings of the induction motor contain only non-triplen odd positive and negative sequence harmonics. Result of which, there will be also non-triplen odd harmonics in stator and rotor currents. One can apply the terms of voltage series to the linear equivalent circuit and obtain the corresponding harmonic term of the current series. Neglecting saturation, the principle of superposition holds and the current produced by each harmonic is independent of the others. Thus each term of Fourier series represents the voltage as a single source as shown in Figure 3.2. Only modification in equivalent impedence network is that at each harmonic frequency, $n\omega_s$ is used to compute the corresponding harmonic current.

3.3.1 Expression for the Stator Current

Let I_1 be the r.m.s. value of the fundamental component of stator current. ϕ_{nq} and ϕ_{nd} are the phase displacement angles between nth stator harmonic voltage and nth stator harmonic current for Q and D axes respectively. K_{nq} and K_{nd} are the ratios of magnitude of stator harmonic current to magnitude of fundamental component of stator current for Q and D axes respectively. So the expression for stator current as per preceeding discussion will be as follows:

$$\bar{i}_{1q} = \bar{i}_{11q} + \bar{i}_{15q} + \bar{i}_{17q} + \ldots\ldots\infty = \bar{I}_{1nq}$$

$$\text{and } \bar{i}_{1d} = \bar{i}_{11d} + \bar{i}_{15d} + \bar{i}_{17d} + \ldots\ldots\infty = \bar{I}_{1nd}$$

where $n = 1,5,7,11,13,17\ldots\ldots\infty$.

So, $\bar{i}_{1nq}$ and $\bar{I}_{1nd}$ can be written to be equal to $I_{1nq}.\sin(n\omega_s t + \phi_{nq})$ and $I_{1nd}.\cos(n\omega_s t + \phi_{nd})$ respectively. Where I_{1nq} and I_{1nd} are the nth harmonic magnitudes of stator current for Q and D axes respectively.

So as per the definition of K_{nq} and K_{nd},

$$K_{nq} = \frac{I_{1nq}}{\sqrt{2}I_1} \quad \text{and} \quad K_{nd} = \frac{I_{1nd}}{\sqrt{2}I_1}$$

Therefore, $i_{1q} = \sqrt{2}I_1 \Sigma K_{nq}.\sin(n\omega_s t + \phi_{nq})$

$$\text{or, } \bar{i}_{1q} = \sqrt{2}I_1 \Sigma K_{nq} \angle \phi_{nq} \tag{3.4a}$$

Similarly, $i_{1d} = \sqrt{2}I_1 \Sigma K_{nd} \cos(n\omega_s t + \phi_{nd})$

$$\text{or, } \bar{i}_{1d} = \sqrt{2}I_1 \Sigma K_{nd} \angle(\phi_{nd} + \pi/2) \tag{3.4b}$$

where $n = 1,5,7,11,13\ldots\ldots\infty$.

3.3.2 Expression for the Rotor Current

Equation (2.22) shows the relation between stator and rotor current phasor having angular frequency ω_s with the machine parameters and speed of the machine, which is as

follows,

$$0 = I_1 . j(\omega_s - \omega_r)\, M + r_2' . I_2' + j(\omega_s - \omega_r) L_{22}' . I_2'$$

implies $$I_2' = \frac{-j\,(\omega_s - \omega_r)\; M . I_1}{r_2' + j(\omega_s - \omega_r) L_{22}'} \qquad (3.5)$$

So, the fundamental component of Q axis rotor current referred to stator -

$$\bar{i}_{21q}' = \frac{-j\,(\omega_s - \omega_r)\; M . i_{11q}}{r_2' + j\,(\omega_s - \omega_r) L_{22}'} \qquad (3.6a)$$

And for nth harmonic Q axis rotor current referred to stator

$$\bar{i}_{2nq}' = \frac{-j\,(n.\omega_s - \omega_r) M . i_{1nq}}{r_2' + j(n.\omega_s - \omega_r) L_{22}'} \qquad (3.6b)$$

$$\text{or } \bar{i}_{2nq}' = \frac{A_n \angle \alpha_{an}}{B_n \angle \alpha_{bn}}\; \bar{i}_{1nq} = C_n \angle \alpha_{cn} . \bar{i}_{1nq}$$

or $\bar{i}_{2nq} = \sqrt{2} I_1\; K_{nq}\; C_n \angle (\alpha_{cn} + \phi_{nq})$ [with the help of eqn.(3.4a)]

So, expression for Q axis rotor current is as follows:

$$\bar{i}_{2q} = \bar{i}_{21q}' + \bar{i}_{25q}' + \bar{i}_{27q}' + \ldots . \infty = \Sigma \qquad \bar{I}_{2nq}'$$

or in other way,

$$i_{2q}' = \sqrt{2} I_1\; \Sigma\; K_{nq} . C_n \sin(\alpha_{cn} + \phi_{nq})$$

$$\text{or} \quad I_{2q}' = \sqrt{2} I_1\; \Sigma\; K_{nq} . C_n \angle(\alpha_{cn} + \phi_{nq}) \qquad (3.7a)$$

Similarly,

$$i'_{2d} = \sqrt{2}I_1 \ \Sigma \ K_{nd}.C_n \ \mathrm{Cos}(\alpha_{cn} + \phi_{nd})$$

or, $$\bar{I}_{2d} = \sqrt{2}I_1 \ \Sigma \ K_{nd}.C_n \angle(\alpha_{cn} + \phi_{nd} + \pi/2) \qquad (3.7b)$$

where $n = 1,5,7,\ldots\ldots\infty$ and $A_n = (n\omega_s - \omega_r)M$; $\alpha_{an} = -\pi/2$

$$B_n^2 = r_2'^2 + \{(n\omega_s - \omega_r)\ L'_{22}\}^2 \ ; \ \alpha_{bn} = \tan^{-1}\{\frac{(n\omega_s - \omega_r)L'_{22}}{r'_2}\}$$

$$C_n = \frac{A_n}{B_n} \ ; \ \alpha_{cn} = (\alpha_{an} - \alpha_{bn}) \ .$$

Now, $$\frac{\text{nth harmonic rotor current}}{\text{nth harmonic stator current}} = \frac{\bar{I}'_{2nq}}{\bar{I}_{1nq}} = \frac{\bar{I}'_{2nd}}{\bar{I}_{1nd}} = C_n \angle \alpha_{cn} \qquad (3.8)$$

So, for fundamental, $$\frac{\bar{I}'_{21q}}{\bar{I}_{11q}} = \frac{\bar{I}'_{21d}}{\bar{I}_{11d}} = C_1 \angle \alpha_{c1} \text{ i.e. } \frac{I'_2}{I_1} = C_1 \ .$$

Thus C_n here represents the ratio of rotor to stator nth harmonic current and α_{cn} is the phase angle difference between these two current phasors.

Figure 3.3 shows the variation of this ratio with fundamental slip for fundamental and other harmonics. At normal frequency of 60 Hz (Figure 3.3(a)), this ratio is almost constant within the fundamental slip ranges of [(0.1) to (1.0)] and [(-0.1) to (-1.0)] owing to the fact that the magnetising reactance is very high compared with the rotor resistance. At synchronous speed, as rotor circuit

is open, fundamental rotor current reduces to zero and C_1 becomes zero. But at very low frequency [Figure 3.3(b)] rotor resistance becomes comparable with magnetising reactance, due to which C_1 never remains constant. It decreases as fundamental slip approaches to zero. For all other harmonics r_2' is always negligible compared to the rotor leakage reactance, even at low frequency as a result of which C_n becomes always constant throughout all speed ranges.

3.3.3 Expression for the Stator Voltage

From equation (2.23), $V_1 = (r_1+j\omega_s L_{11})I_1+j\omega_s M I_2'$ which implies that stator voltage having angular frequency ω_s is related with stator and rotor current along with machine parameters. Therefore, fundamental component of Q axis stator voltage

$$\bar{v}_{11q} = (r_1 + j\omega_s L_{11})\ \bar{I}_{11q} + j\ \omega M\ \bar{I}'_{21q}$$

For nth harmonic, expression becomes as follows:

$$\bar{v}_{1nq} = (r_1 + jn\omega_s L_{11})\bar{I}_{1nq} + j\ n\omega_s M\ \bar{I}'_{2nq} \qquad (3.9a)$$

$$= (F_n \angle \alpha_{fn} + H_n \angle \alpha_{hn})\ \bar{I}_{1nq}$$

or $$v_{1nq} = Z_n \angle\alpha_{zn} \cdot \bar{I}_{1nq} \qquad (3.9b)$$

where $F_n^2 = r_1^2 + (n\omega_s L_{11})^2;\ \alpha_{fn} = \tan^{-1}\left(\frac{n\omega_s L_{11}}{r_1}\right)$

$G_n = n\omega_s M;\ \alpha_{gn} = \pi/2;\ H_n = G_n . C_n;\ \alpha_{hn} = (\alpha_{gn} + \alpha_{cn})$

and $Z_n^2 = Z_{cn}^2 + Z_{sn}^2;\quad Z_{cn} = F_n \cos\alpha_{fn} + H_n \cos\alpha_{hn}$

$\alpha_{zn} = \tan^{-1}(Z_{sn}/Z_{cn});\ Z_{sn} = F_n \sin\alpha_{fn} + H_n \sin\alpha_{hn}$

Equation 3.9(b) can be rewritten as

$$\bar{v}_{1nq} = \sqrt{2}I_1 . K_{nq} . Z_n \angle (\alpha_{zn} + \phi_{nq})$$

Therefore, $\bar{v}_{1q} = \sqrt{2}I_1 \sum_{n=1,5,7}^{\infty} K_{nq} . Z_n \angle (\alpha_{zn} + \phi_{nq})$ (3.10a)

and $v_{1d} = \sqrt{2}I_1 \sum_{n=1,5,7}^{\infty} K_{nd} . Z_n \angle (\alpha_{zn} + \phi_{nd} + \pi/2)$ (3.10b)

Again $\bar{v}_{1nq}/\bar{i}_{1nq} = Z_n \angle \alpha_{zn}$ which represent equivalent impedance for nth harmonic voltage $\bar{v}_{1nq}$ looking from stator side with rotor shorted.

3.4 METHOD OF ANALYSIS WITH RESULTS

From equation 3.3(a) and 3.10(a)

$$v_{1q} = \frac{2V_{dc}}{\pi} \Sigma \sin(n\omega_s t)/n = \sqrt{2}I_1 \Sigma K_{nq} . Z_n \angle (\alpha_{zn} + \phi_{nq})$$

where $n = 1, 5, 7, 11, 13, \ldots\ldots \infty$,

or, $\bar{v}_{1nq} = \frac{2V_{dc}}{n\pi} \angle 0° = \sqrt{2}I_1 . K_{nq} . Z_n \angle (\alpha_{zn} + \phi_{nq})$.

Now comparing the magnitude and phase angle of the nth harmonic voltage,

$$\frac{2V_{dc}}{n\pi} = \sqrt{2}I_1 . K_{nq} . Z_n \qquad (3.11a)$$

and, $\alpha_{zn} + \phi_{nq} = 0$ (3.11b)

So for fundamental quantities

$$\frac{2V_{dc}}{\pi} = \sqrt{2}I_1 . K_{1q} . Z_1 \qquad (3.12a)$$

and $\alpha_{z1} + \phi_{1q} = 0$ (3.12b)

Now dividing equation 3.11(a) by 3.12(a) gives

$$\frac{1}{n} = \left(\frac{K_{nq}.Z_n}{K_{1q}.Z_1}\right) \quad \text{or} \quad K_{nq} = \left(\frac{K_{1q}.Z_1}{n.Z_n}\right) \qquad (3.13a)$$

and from equation 3.11(b) $\phi_{nq} = -\alpha_{zn}$ and equation 3.12(b) $\phi_{1q} = -\alpha_{z1}$.

Again $I_{11q} = \sqrt{2}I_1$ therefore $K_{1q} = \dfrac{I_{11q}}{\sqrt{2}I_1} = 1$.

So equation 3.13(a) reduces to $K_{nq} = \left(\dfrac{Z_1}{nZ_n}\right)$ (3.13b)

Expression for Z_n has already derived which is dependent upon the machine parameters and angular harmonic frequency $(n\omega_s)$ of the input voltage. Z_1 can be found by substituting $n = 1$ in the expression of Z_n. In the same way, K_{nq} for $n = 1,5,7,11....$etc. can be known from the equation 3.13(b). In the same fashion K_{nd} can be found for various harmonics.

Expression for α_{zn} has also been derived earlier. So putting $n = 1$ in that expression of α_{zn}, α_{z1} can be found. So just by changing the value of n, ϕ_{nq} can also be determined for $n = 1,5,7,....$etc. Same method is also valid for getting

ϕ_{nd} for all existing harmonics.

3.4.1 Effect of Operating Frequency and Machine Parameters on Current Harmonics

Absolute values of K_{nq} or K_{nd} for fundamental and other three harmonics are plotted in Figure 3.4(a) for normal operating frequency of 60 Hz. The plot shows that close to synchronous speed, harmonic currents compared with fundamental become appreciablly high and at synchronous speed due to minimum value of fundamental current, the value of K_{nq} or K_{nd} reaches its maximum. It is found that K_{5q}, K_{7q} and K_{11q} become 77.11%, 39.44% and 16.02% respectively at zero fundamental slip (Table 3.1).

Again in Figure 3.4(b), the same characteristics are shown making r_1 to be ideally zero. As equivalent harmonic reactance strongly predominates over r_1, only slight changes are obtained in this characteristics as compared with Figure 3.4(a) to make it almost symmetric about the motoring and generating region.

At low frequencies, harmonic currents are comparable with fundamental r.m.s. current. So ratio of harmonic to fundamental current increases as operating frequency decreases irrespective of all harmonics, except close to synchronous speed (fundamental slip = $\pm$ 0.10). But with the decrease of frequency, fundamental current does not decrease at a rapid rate as speed approaches to synchronous speed. As a consequence

of which harmonic content in current becomes less at zero fundamental slip, as compared with normal frequency. Figures 3.4(c) and 3.4(d) show these behaviours for inverter operating frequency of 30 Hz and 10 Hz.

Again at low frequencies, r_1 is comparable with the equivalent reactance. Thus absence of r_1 makes this ratio to be still higher than for normal low frequencies which are shown in Figures 3.4(f) and 3.4(h) [as compared with 3.4(e) and 3.4(g) respectively].

3.4.2 Effect of Operating Frequency and Machine Parameters on Power Factor

(a) Fundamental Power Factor

It is defined as the cosine of the phase displacement angle between fundamental component of input voltage and fundamental component of input current. Table 3.2 shows the variation of fundamental and harmonic power factors with fundamental slip for different values of inverter operating frequency.

In Figure 3.5(a), fundamental power factor increases as speed of the motor increases due to increase in effective rotor resistance in the positive fundamental slip region. It reaches its maximum at a particular speed which is dependent on machine parameters and inverter operating frequency. Beyond this speed rotor circuit becomes ineffective and only effect of magnetising branch comes into picture along with the stator

parameters for which fundamental power factor approaches zero value. Again beyond synchronous speed it reaches another peak for the similar reason as above. But beyond the peak in the generating region it approaches approximately zero value as the value of stator resistance gets cancelled with the effective resistance introduced by the magnetising branch and rotor circuit irrespective of inverter operating frequencies. It is also found that, gradual decrease in frequency results in improved power factor in both the regions as inductive reactance gets reduced.

Effect of stator resistance on fundamental power factor for varying operating frequencies are shown in Figures 3.5(b) to 3.5(e). Stator resistance ideally zero [Figure 3.5(e)] makes the characteristics perfectly symmetrical about the two regions of the fundamental slip. It is also observed that the product of stator resistance and the magnitude of fundamental slip at zero fundamental power factor is more or less constant under all operating frequencies. But this property does not hold for very high value of stator resistances which are shown in Figures 3.5(d) and 3.5(e). Also for high values of stator resistance, this power factor approaches unity under all frequencies, especially in the motoring region.

Figure 3.5(f) shows the fundamental power factor characteristics for low frequencies along with the normal

frequency. It is observed that at very low frequencies and at specially low speed region, this power factor becomes high as compared with normal frequency. But as speed approaches to synchronous speed, rotor circuit tends to be open circuited, due to which fundamental power factor gets reduced. The above characteristics has been drawn again in Figure 3.5(g) with a special case of r_1 to be ideally zero, where absence of r_1 leads to low power factor throughout all speed ranges due to predominant inductive effect in the equivalent circuit.

Figure 3.5(h) shows fundamental power factor versus fundamental slip characteristics and depicts the effect of rotor time constants. It is observed that depending upon the increase or decrease in rotor resistance, the zero power factor point shifts towards high or low value of fundamental slip in the generating region. Besides, insertion of rotor resistance results in the improvement of fundamental power factor in both the regions.

(b) Harmonic Power Factor

The effect of inverter operating frequency and machine parameters on the harmonic power factor is shown in Figures 3.6(a)-(h). The following points can be noted.

(i) Positive sequence seventh harmonic power factor increases with the motor speed whereas negative

sequence fifth and eleventh harmonic power factors decrease as fundamental slip of the motor decreases. The slip due to harmonic (S_n) is given by

$$S_n = \frac{n-1+S}{n} \text{ (for positive sequence harmonics)}$$

and

$$S_n = \frac{n+1-S}{n} \text{ (for negative sequence harmonics).}$$

where S is the fundamental slip.

Thus as motor speed increases S_n decreases or increases [Table 3.3] depending upon the sequence of the harmonics. Therefore, effective rotor resistance of the motor increases and decreases respectively for positive and negative sequence harmonics. This results in increase or decrease in harmonic factor as motor speed changes.

(ii) It is clear from the Figures that the harmonic power factor decreases as the order of harmonic increases. This is easily explained as at higher frequencies the inductances dominate in the machine equivalent circuit.

(iii) For the same reason as in (ii), the harmonic power factor decreases as inverter operating frequency is increased.

(iv) If the stator resistance is reduced to be ideally zero in the model, the harmonic power factor goes down.

any constant inverter operating frequency, the peak current in the generating region will be greater than the starting current of the induction motor due to finite stator resistance.

For ideally r_1 to be zero, I_1 versus slip characteristics in the generating region takes the mirror image of the I_1 versus slip characteristics in the motoring region about the current axis at zero slip, which is shown in Figure 3.7(b).

In Figure 3.7(c), effect of low frequency on I_1 versus slip characteristics is shown, taking standstill current of the motor at any operating frequency to be 1.0 p.u. It is observed that as speed increases from zero, the low frequency current decreases and after a particular speed of the motor this current increases till synchronous speed. The point of transition is dependent mainly on the operating frequency. Even at 1 Hz frequency, current close to synchronous speed becomes higher than the standstill motor current. But this observation is not valid where r_1 is made ideally to be zero [Figure 3.7(d)].

Figure 3.7(e) shows the effect of varying rotor time constant of the motor on fundamental stator r.m.s. current versus slip characteristics. Starting current reduces with the insertion of rotor resistance. Here also I_1 is found to be independent of rotor time constant at very close to synchronous speed of the motor. The speed where I_1 attains

its peak in the negative slip region, shifts away from zero slip point with the increase in rotor resistance.

3.5.2 Effect of Operating Frequency and Machine Parameters on Harmonic Stator r.m.s. Current

Variation of harmonic currents with the fundamental slip is depicted in Figure 3.8 for 60 Hz, 3 Hz and 1 Hz. It is found that all r.m.s. values of harmonic current ($I_n = K_{nq} \times I_1$) become nearly constant at normal operating frequency of 60 Hz [Figure 3.8(a)] for any speed operation. The value of this constant current depends on the order of harmonic. Motor parameters do not affect much on these characteristics.

But with the decrease of operating inverter frequency harmonic currents no more remain constant. Depending upon the harmonic contents at different speed [Figures 3.4(e) and 3.4(g)] and r.m.s. values of the fundamental current, harmonic currents change more or less linearly with the fundamental slip of the motor which is shown in Figures 3.8(b) and 3.8(c) for 3 Hz and 1 Hz respectively.

3.5.3 Effect of Inverter Operating Frequency and Motor Speed on Instantaneous Stator Current of the Induction Motor Under Consideration

Figures 3.9 to 3.12 show the stator current waveforms including the predominant existing harmonics for varying speed and inverter frequency operation of the induction motor.

Pattern and peak of the waveshape change depending upon the following:

(i) Fundamental power factor (refer to Figure 3.5).

(ii) Harmonic power factor (refer to Figure 3.6).

(iii) Fundamental r.m.s. current (refer to Figure 3.7).

(iv) Harmonic r.m.s. current (refer to Figure 3.8).

At normal frequency of 60 Hz, current waveforms [Figures 3.9(a), 3.9(b)] are not distorted much from the sinusoidal, particularly in the low speed regions. But as speed approaches synchronous speed, due to predominant effect of current harmonics [Figure 3.4(a)] the waveforms shown in Figures 3.9(c) to 3.9(f) change their envelope abruptly as compared with low speed current waveforms.

Low frequency instantaneous stator current waveforms are drawn in Figures 3.11 to 3.12 for studying the effect of current harmonics. It is observed that with the proportionate change in speed (as was made for normal frequency), the low frequency waveforms do not change their shape much especially close to synchronous speed due to very little change in their harmonic contents.

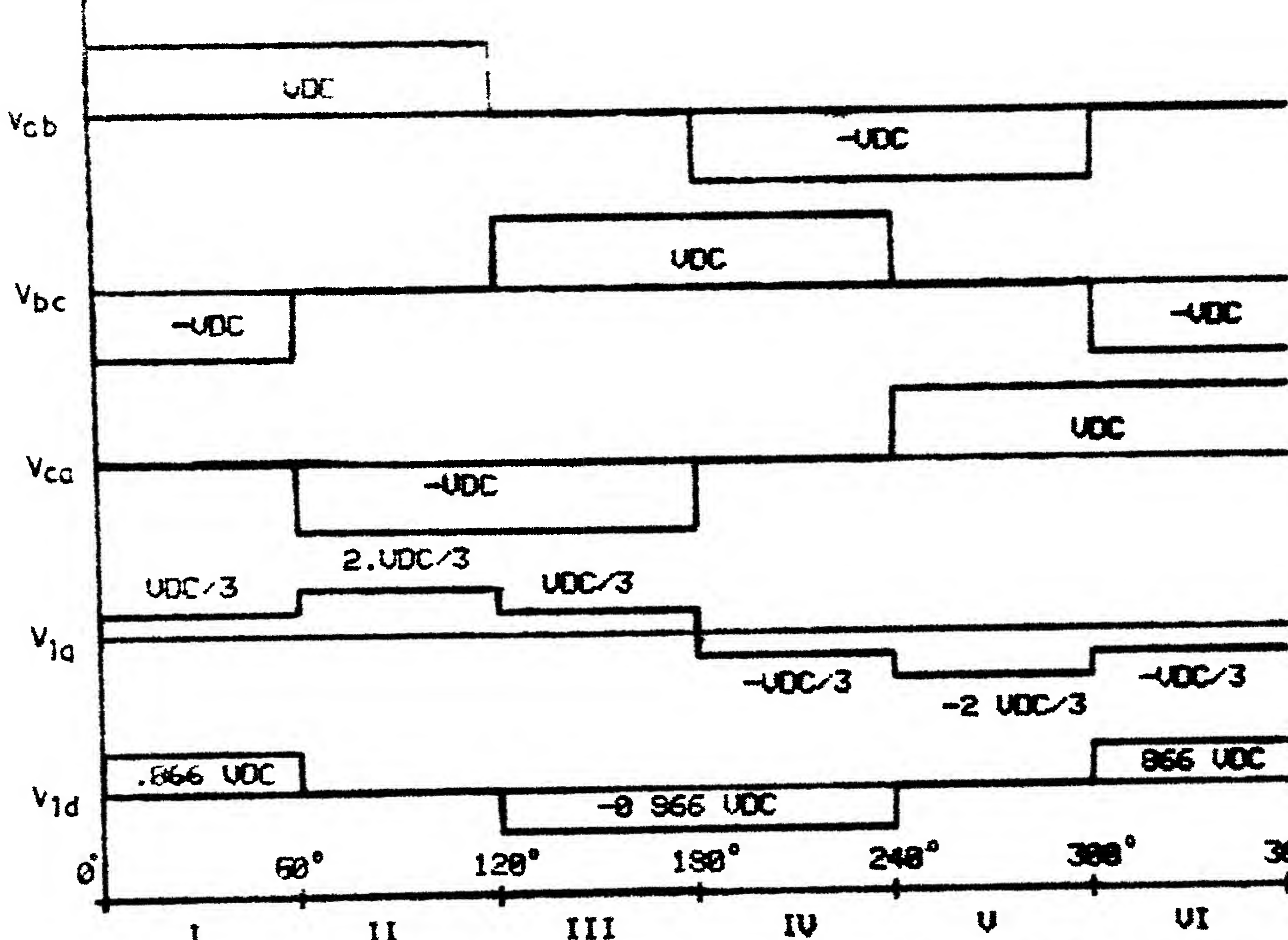
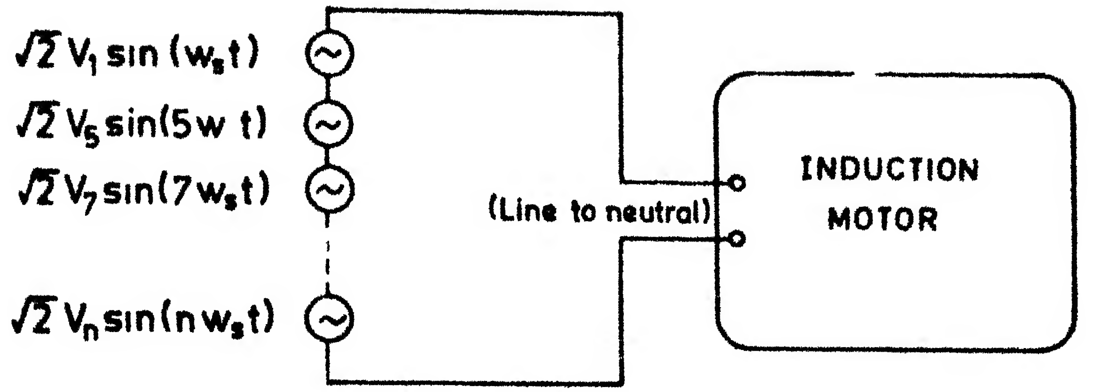
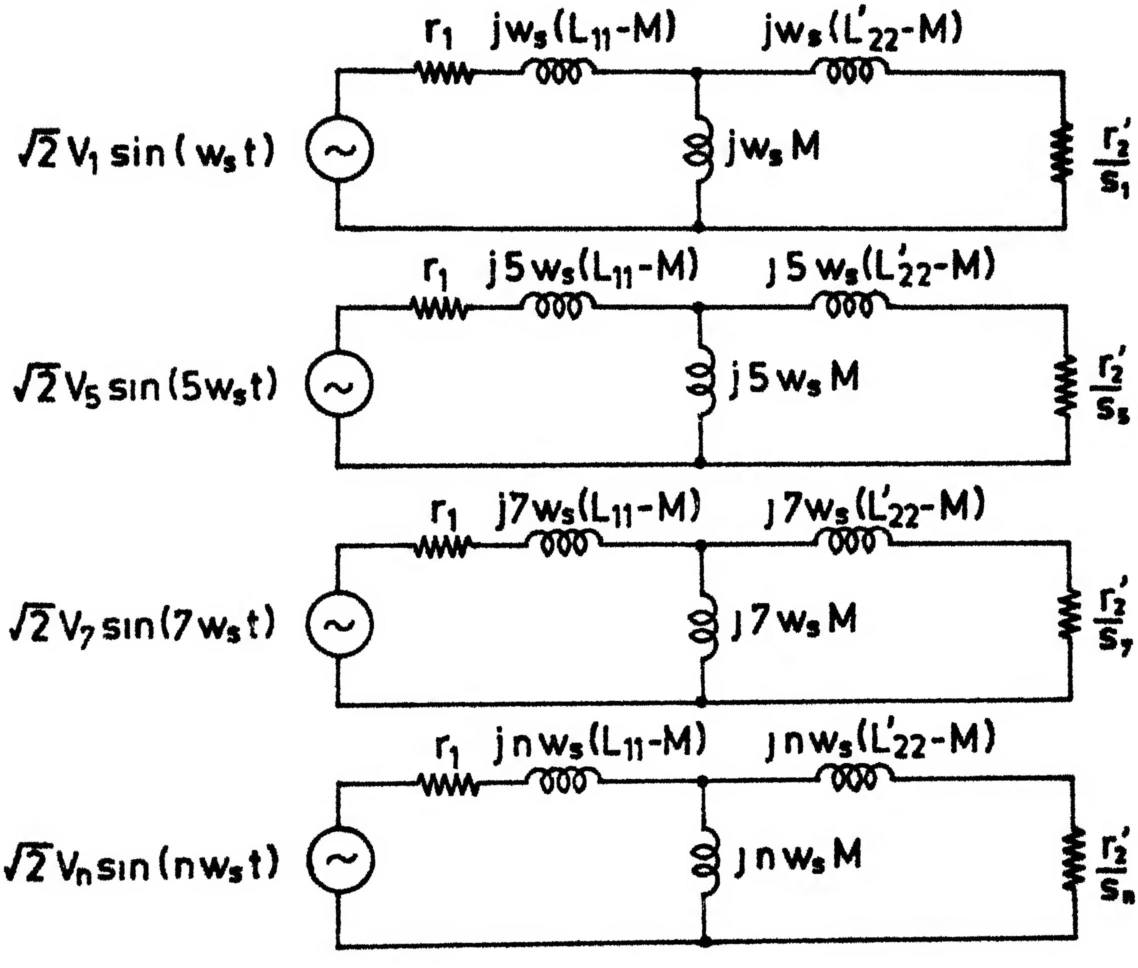


FIG 3 1 THREE PHASE LINE & D/Q STATOR VOLTAGES FOR 180 DEGREE CONDUCTION



(a)



(b)

Fig 3.2 (a) Non sinusoidal excitation of induction motor (per phase)

(b) Equivalent circuit for induction motor with non-sinusoidal excitation

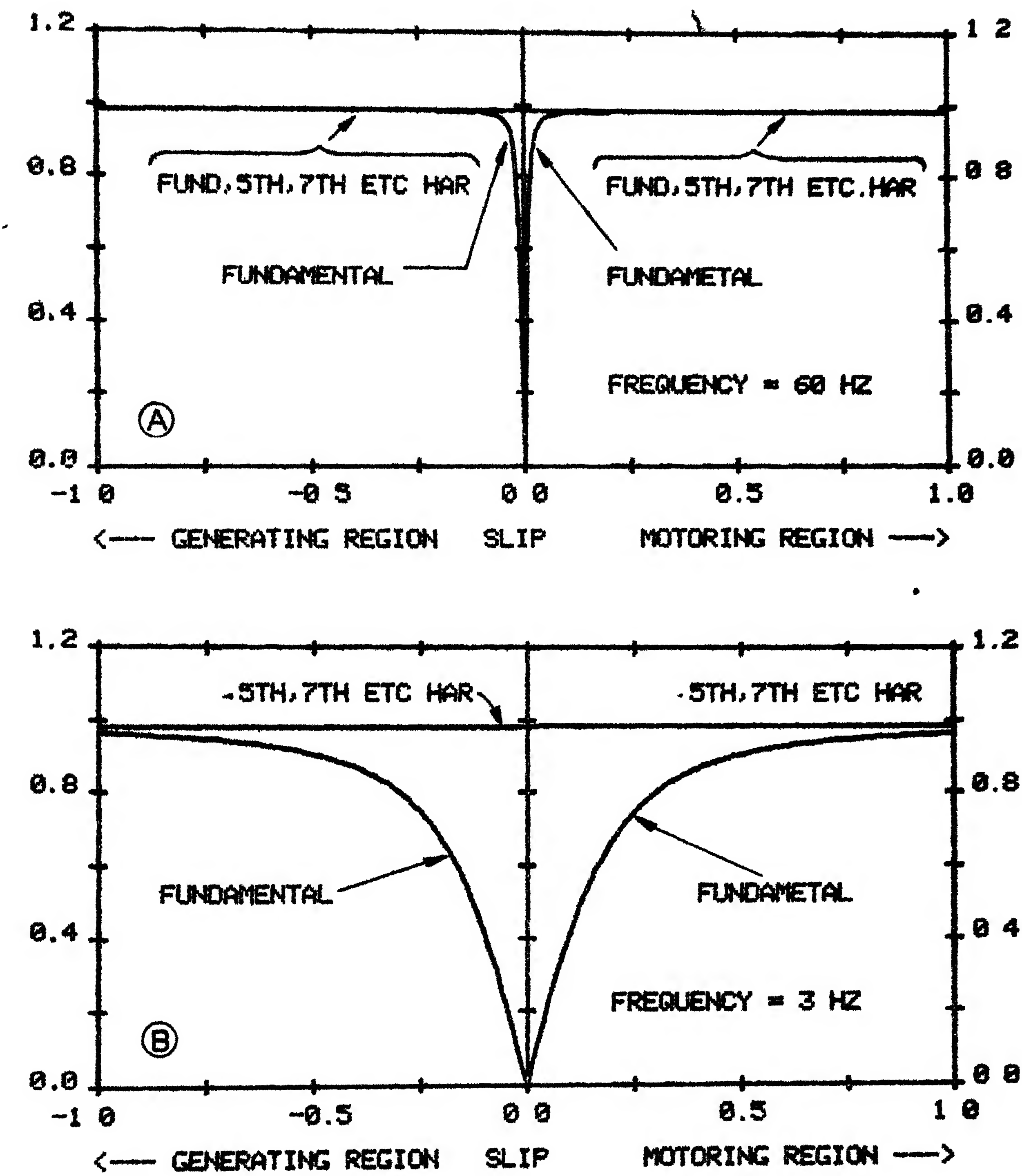


FIG 3 3 RATIO OF HARMONIC ROTOR CURRENT TO HARMONIC STATOR CURRENT VERSUS FUNDAMENTAL SLIP CHARACTERISTICS

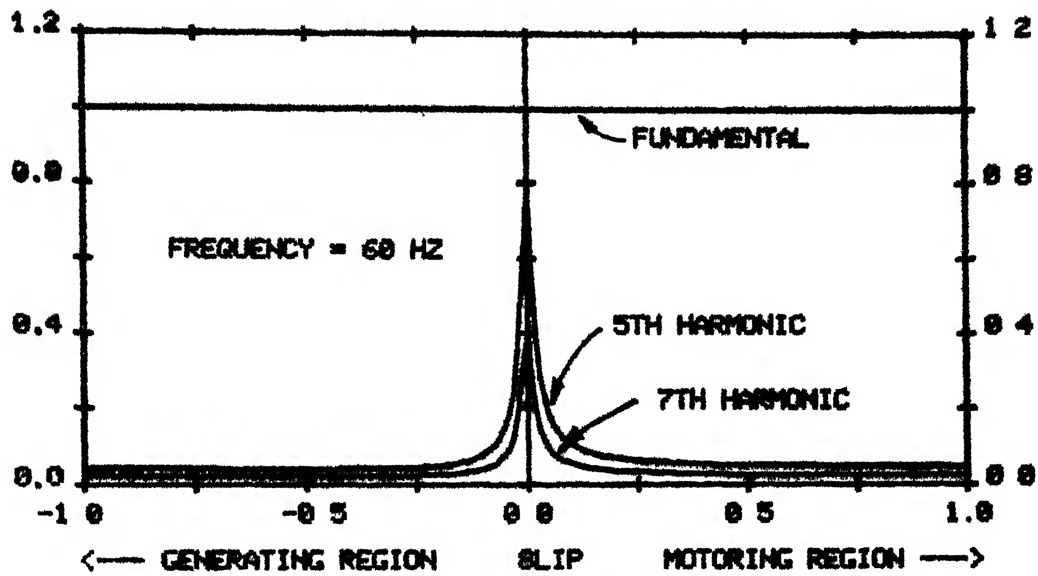


FIG 3 4(A) RATIO OF HARMONIC CURRENT TO FUNDAMENTAL CURRENT VERSUS SLIP

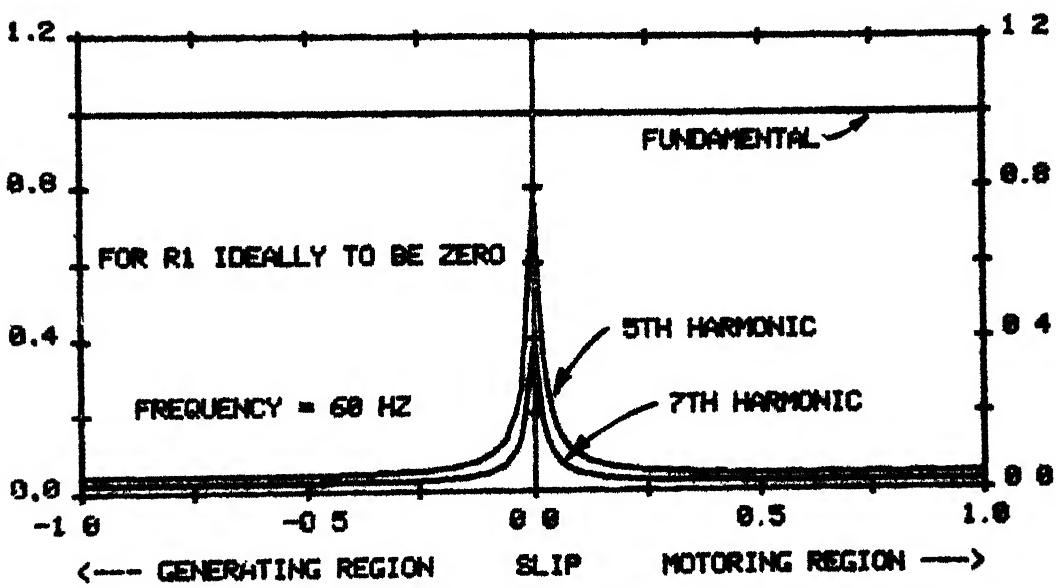


FIG 3 4(B) RATIO OF HARMONIC CURRENT TO FUNDAMENTAL CURRENT VERSUS SLIP

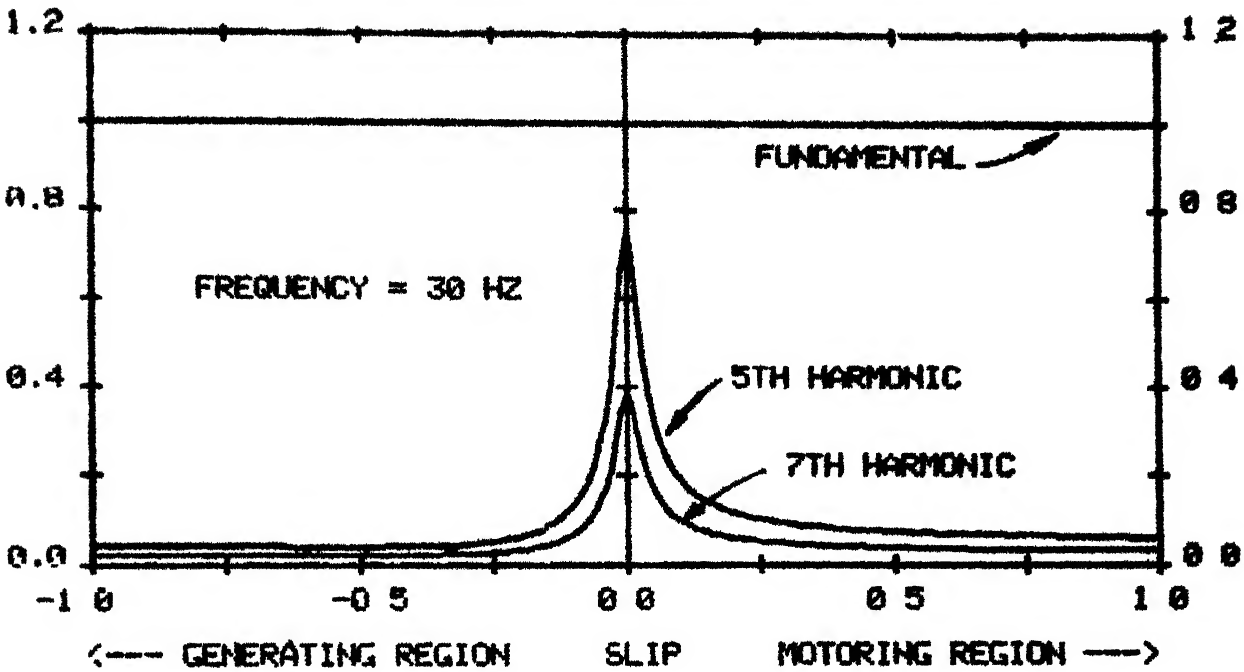


FIG 3 4(C) RATIO OF HARMONIC CURRENT TO FUNDAMENTAL CURRENT VERSUS SLIP

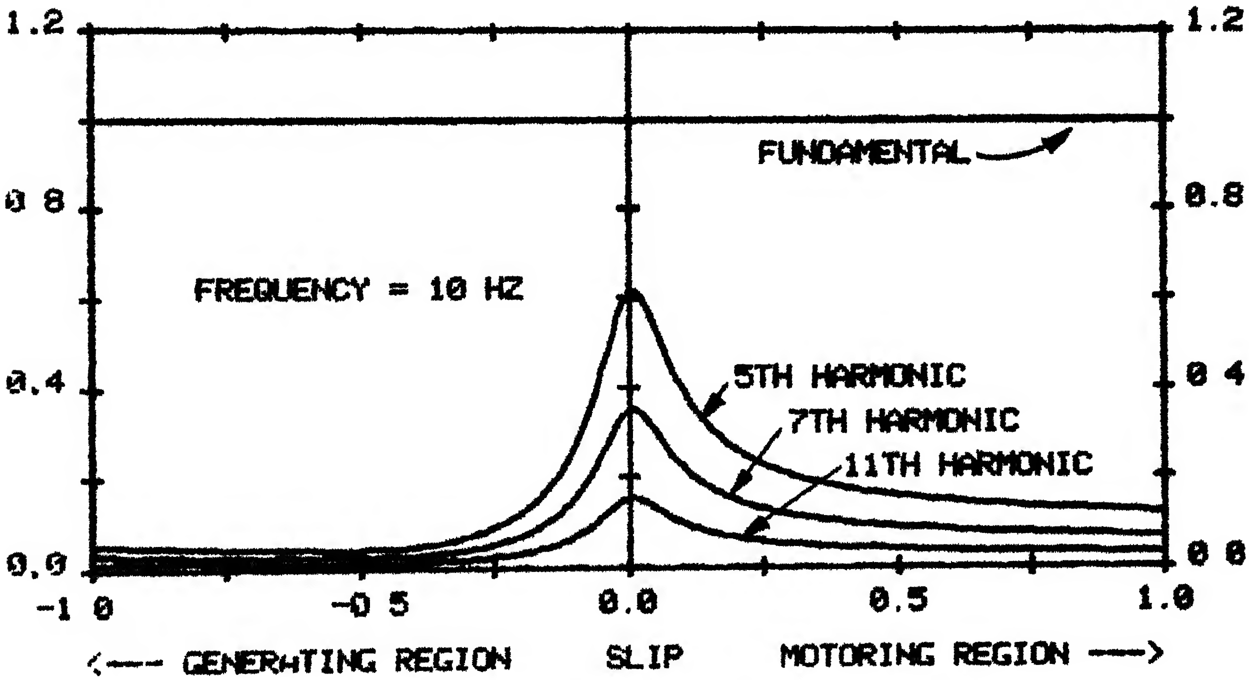


FIG 3 4(D) RATIO OF HARMONIC CURRENT TO FUNDAMENTAL CURRENT VERSUS SLIP

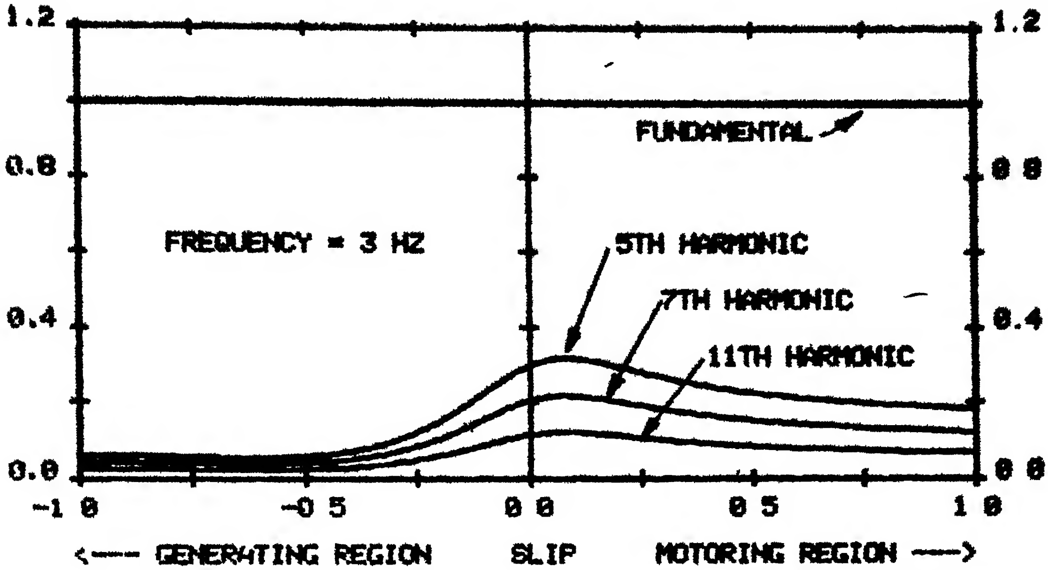


FIG 3 4(E) RATIO OF HARMONIC CURRENT TO FUNDAMENTAL CURRENT VERSUS SLIP

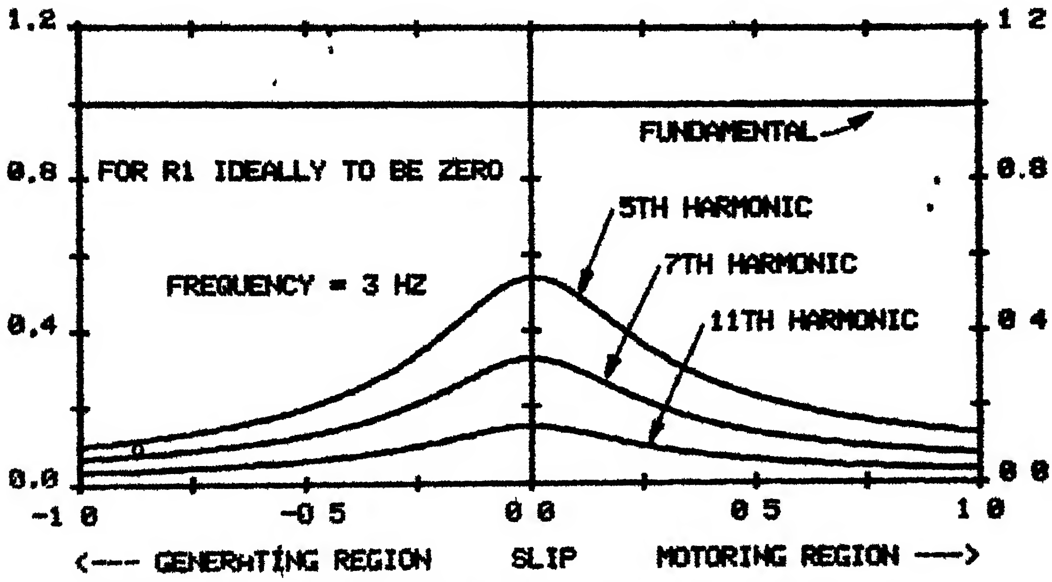


FIG 3 4(F) RATIO OF HARMONIC CURRENT TO FUNDAMENTAL CURRENT VERSUS SLIP

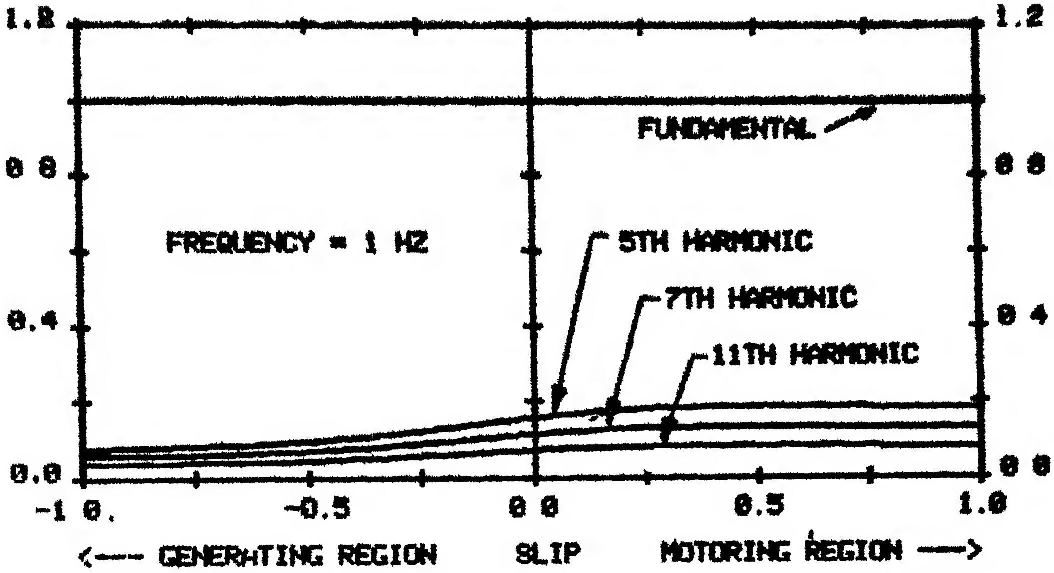


FIG 3 4(G) RATIO OF HARMONIC CURRENT TO FUNDAMENTAL CURRENT VERSUS SLIP

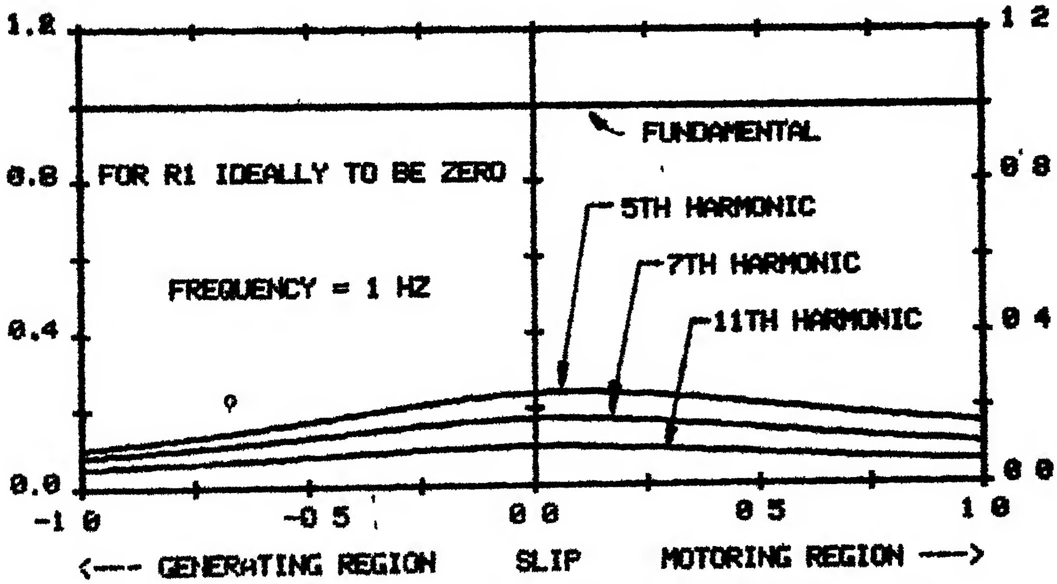


FIG 3 4(H) RATIO OF HARMONIC CURRENT TO FUNDAMENTAL CURRENT VERSUS SLIP

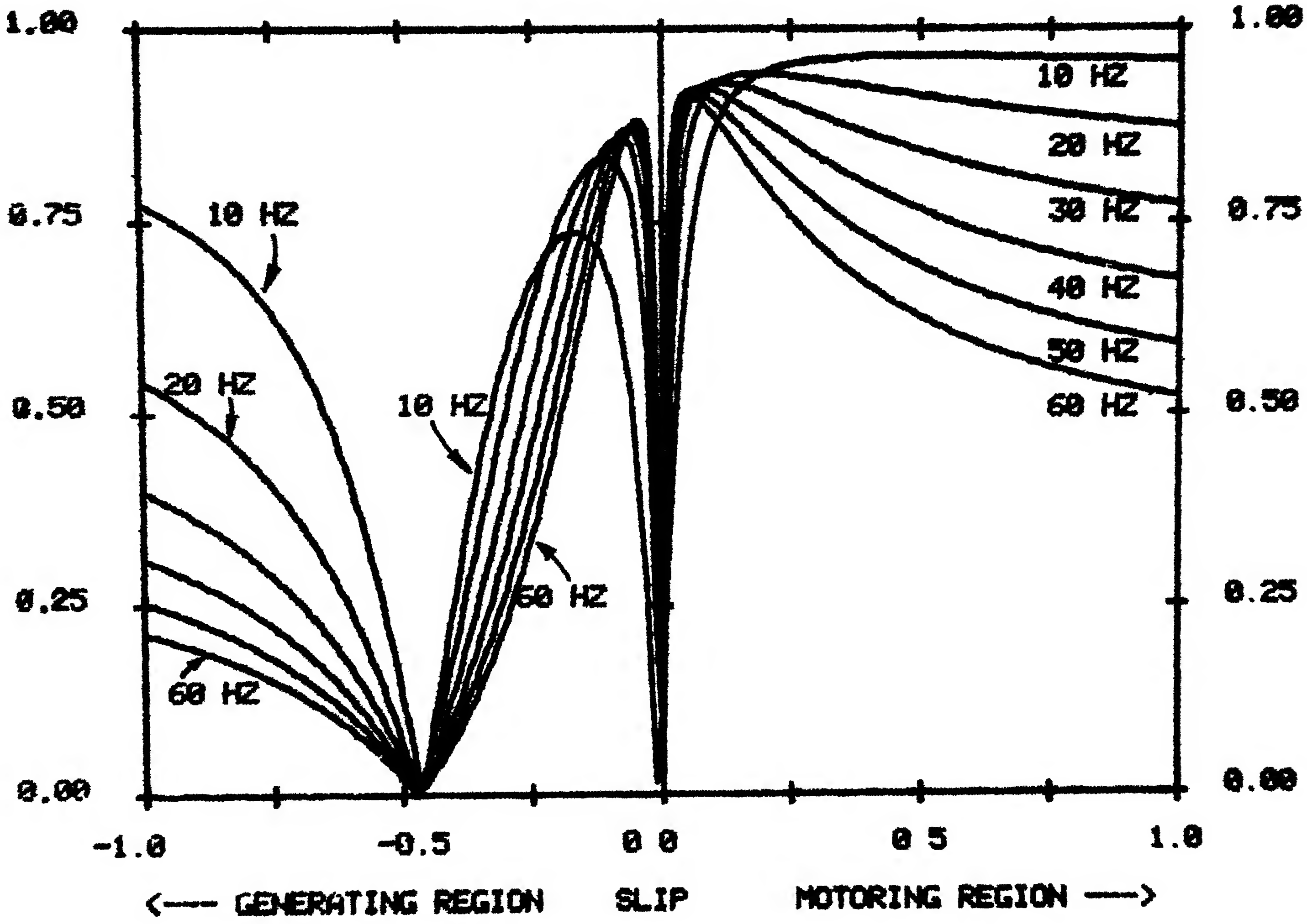


FIG 3 5(A) VARIATION OF FUNDAMENTAL POWER FACTOR WITH SLIP

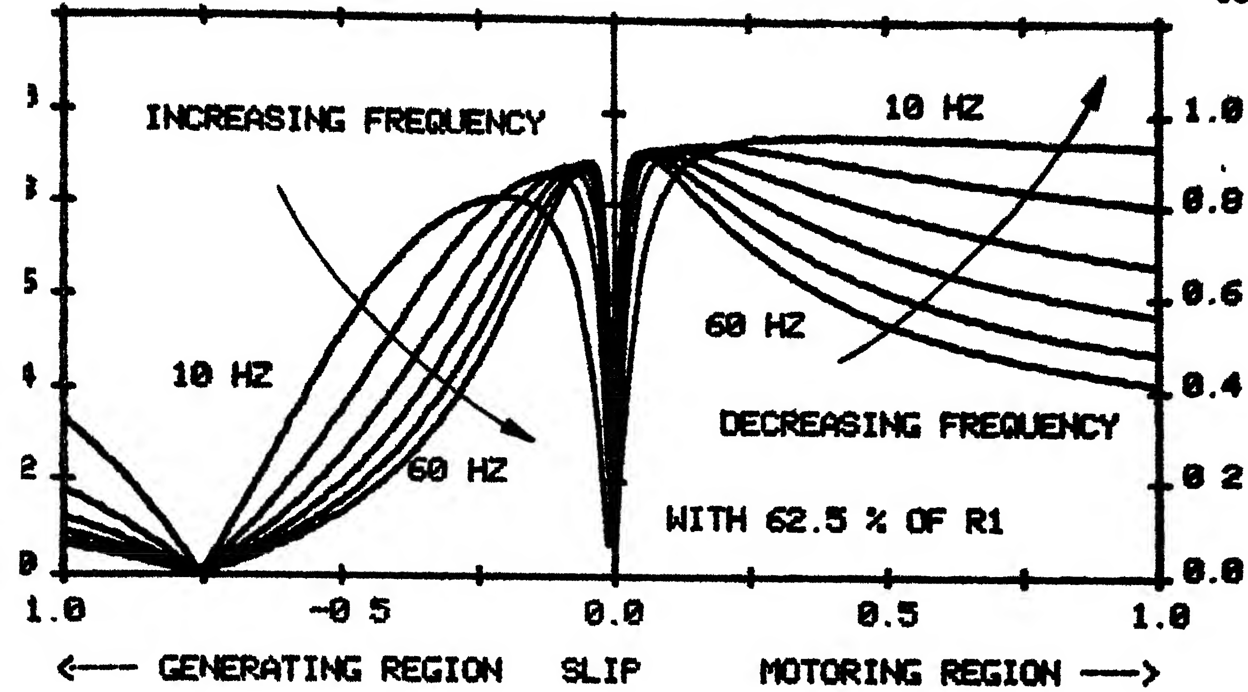


FIG 3,5(B) VARIATION OF FUNDAMENTAL POWER FACTOR WITH SLIP

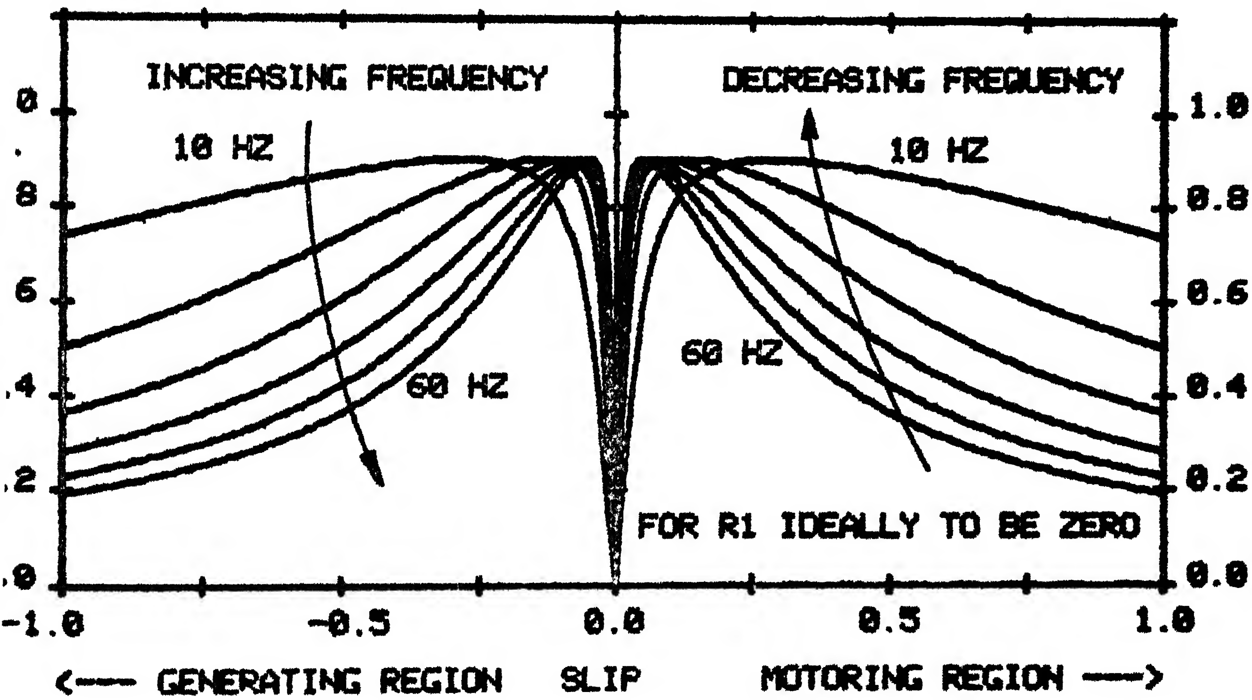


FIG 3 5(C) VARIATION OF FUNDAMENTAL POWER FACTOR WITH SLIP

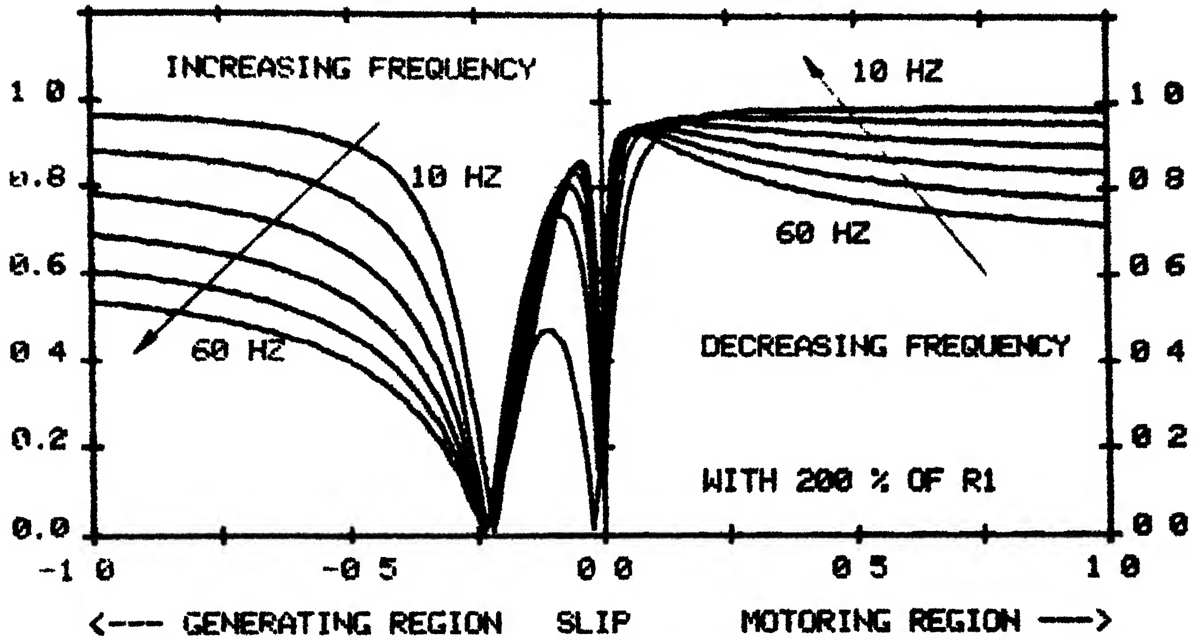


FIG 3 5(D) VARIATION OF FUNDAMENTAL POWER FACTOR WITH SLIP

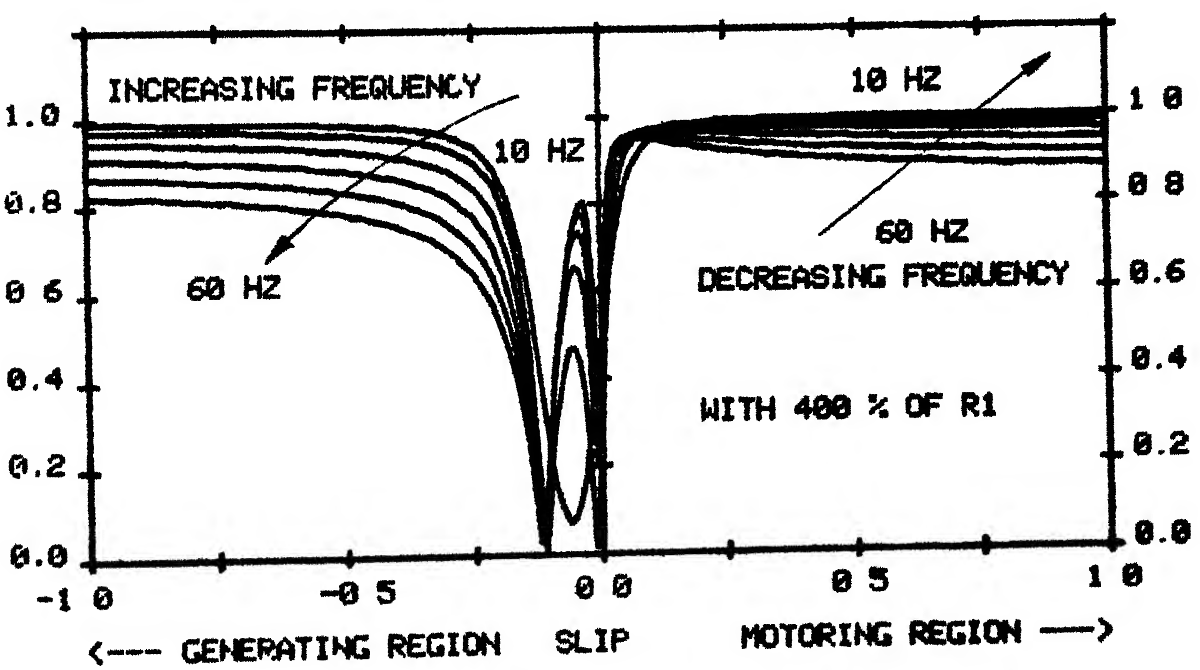


FIG 3 5(E) VARIATION OF FUNDAMENTAL POWER FACTOR WITH SLIP

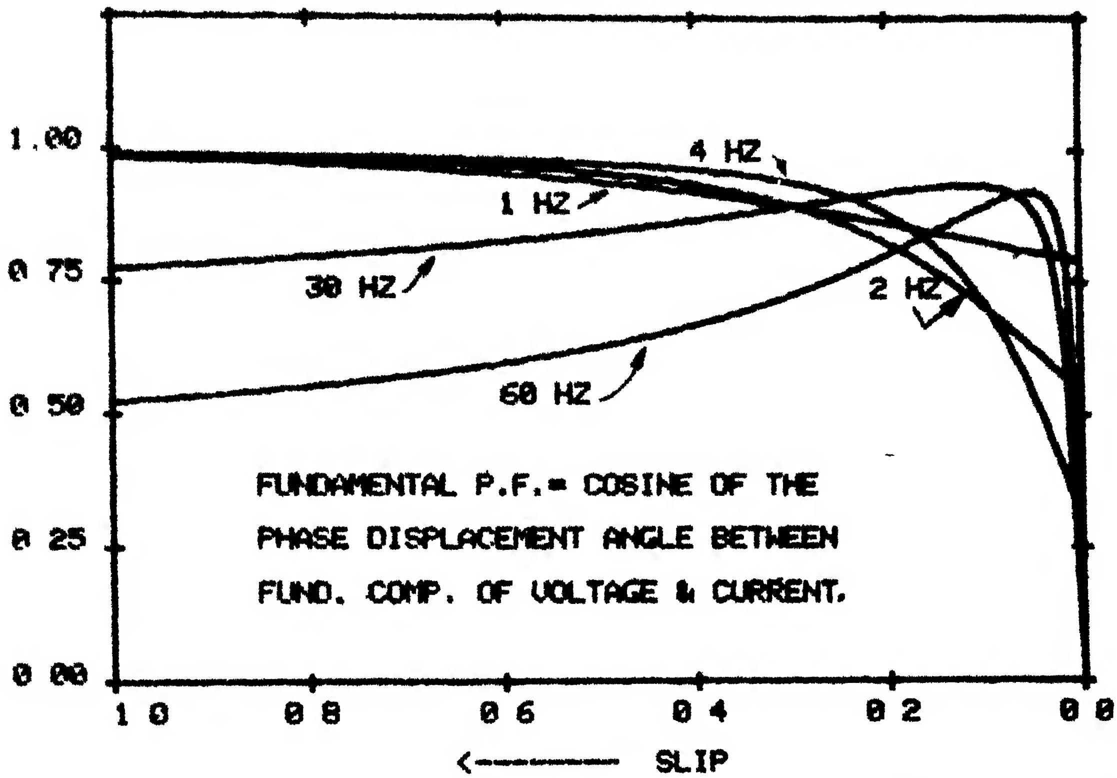


FIG 3.5(F) VARIATION OF FUNDAMENTAL POWER FACTOR WITH SLIP

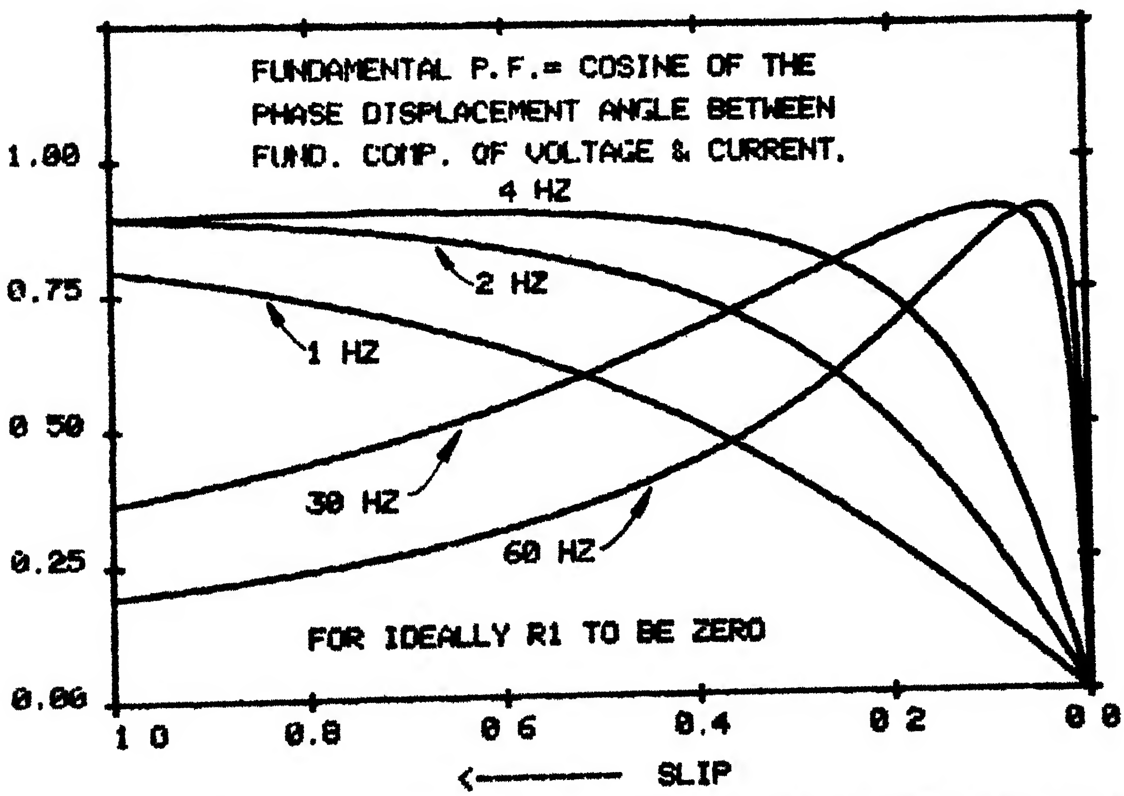


FIG 3 5(G) VARIATION OF FUNDAMENTAL POWER FACTOR WITH SLIP

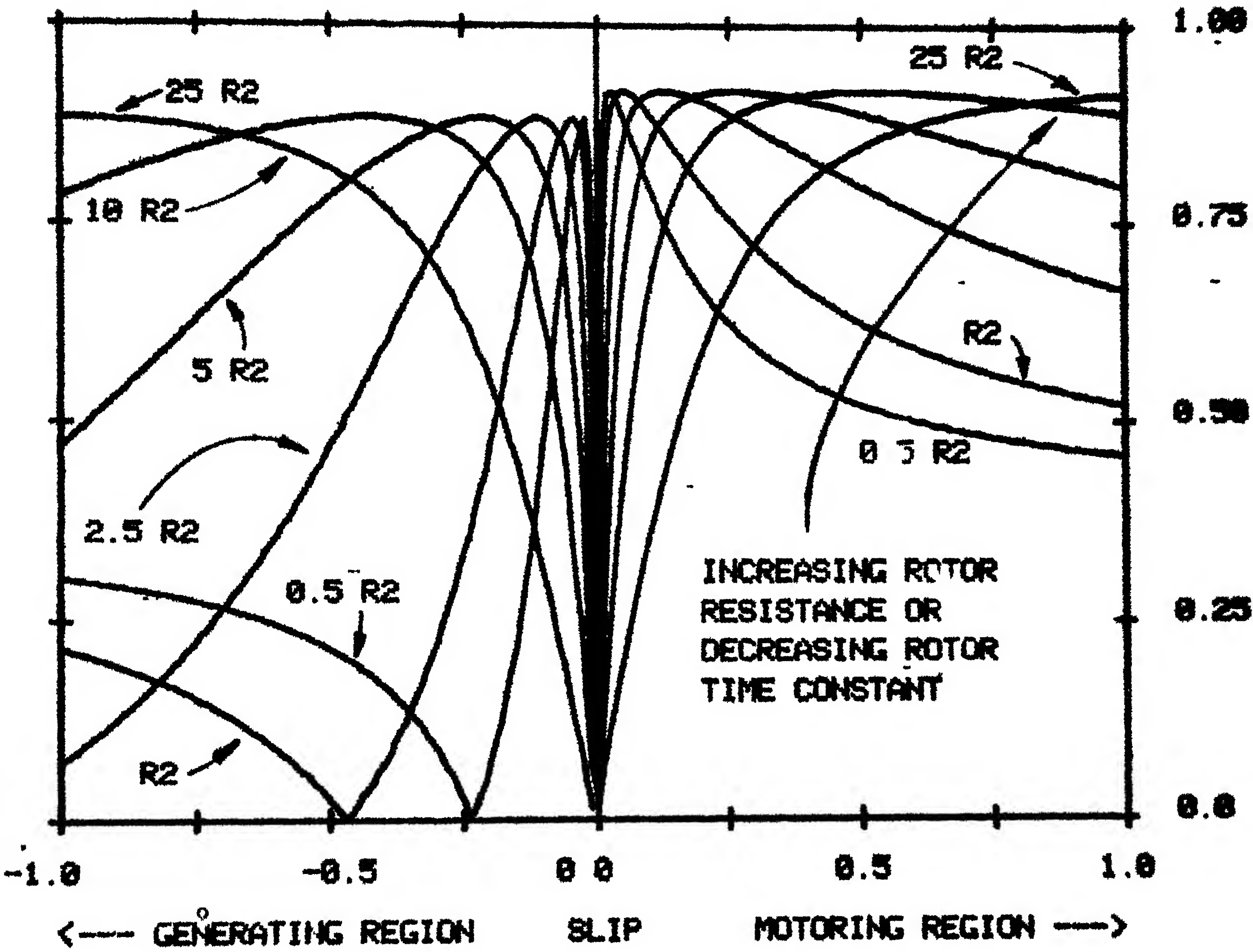


FIG 3 5(H) VARIATION OF FUNDAMENTAL POWER FACTOR WITH SLIP

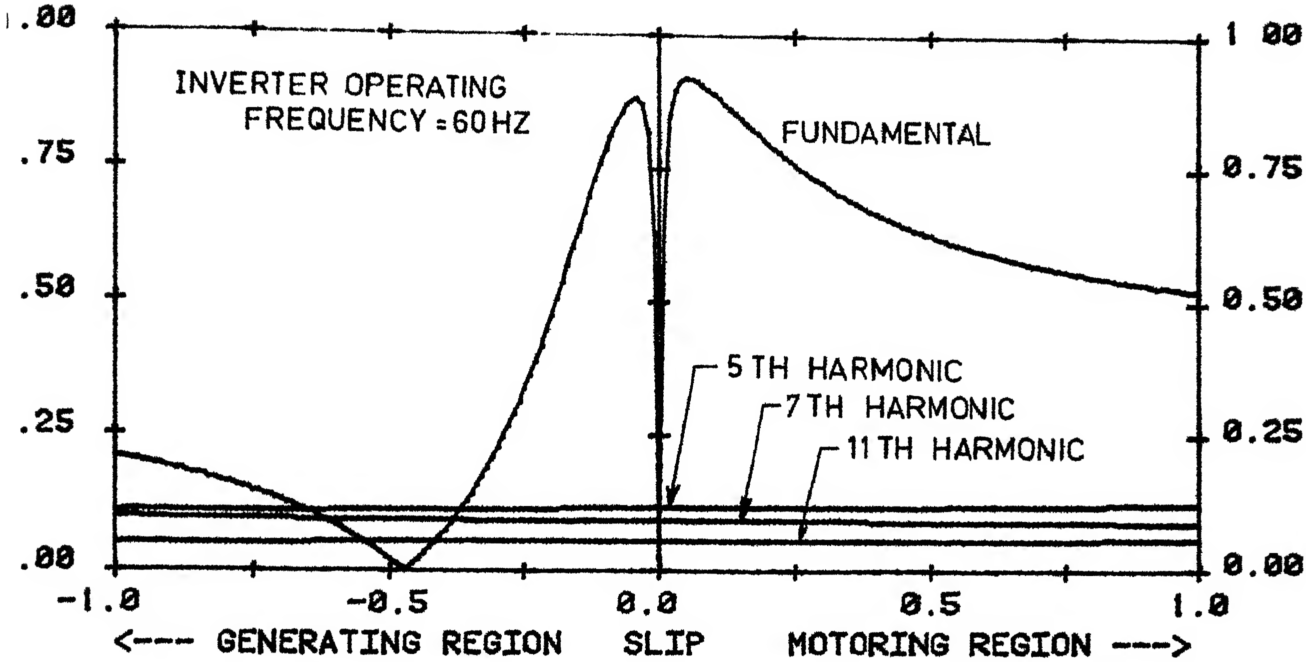


FIG 3.8(A) VARIATION OF HARMONIC POWER FACTOR WITH FUNDAMENTAL SLIP

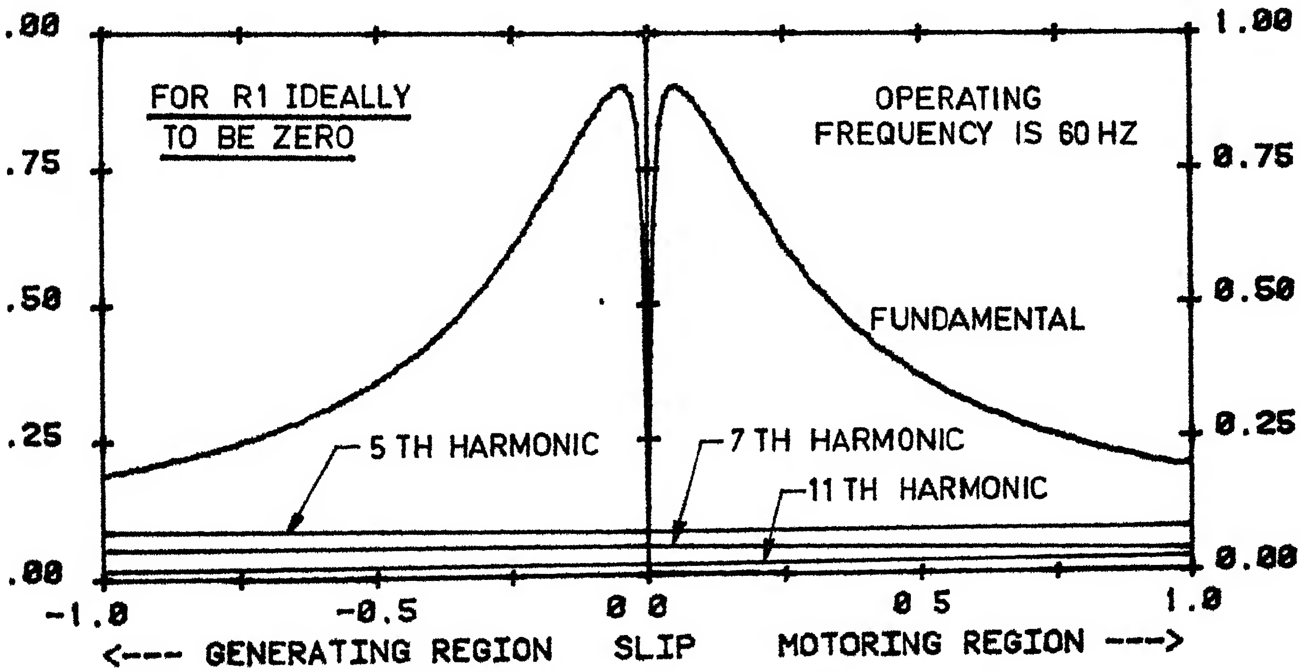


FIG 3 8(B) VARIATION OF HARMONIC POWER FACTOR WITH FUNDAMENTAL SLIP.

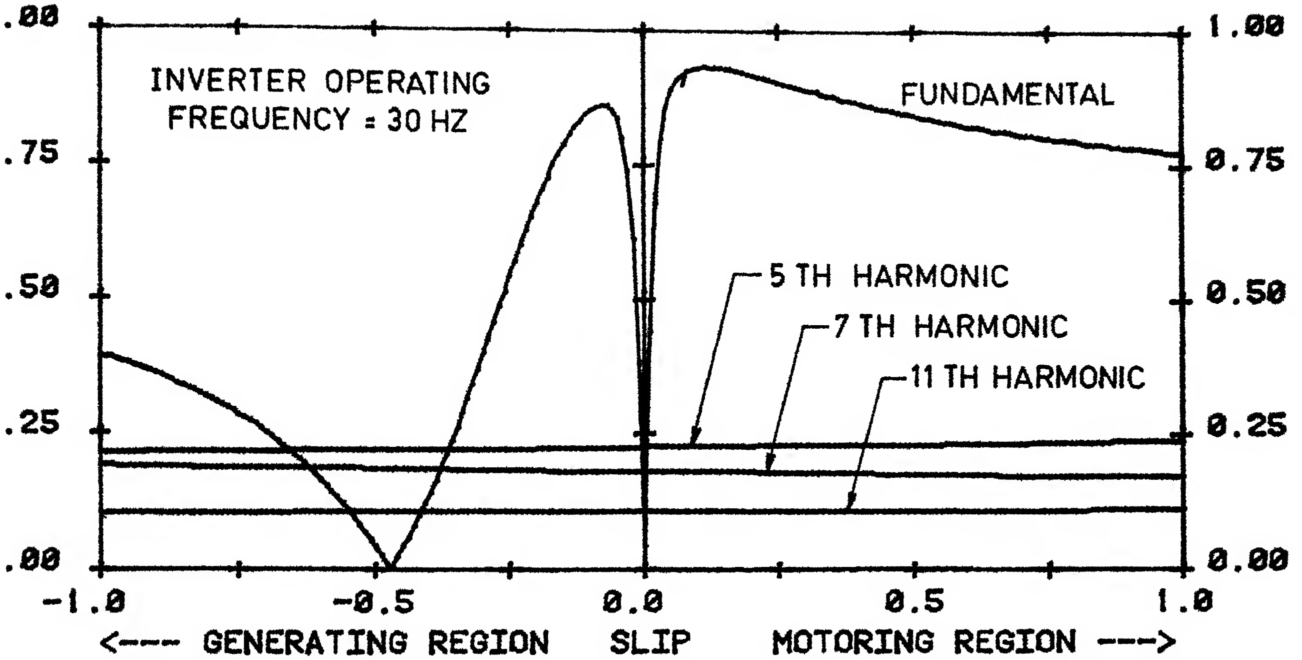


FIG 3.6(C) VARIATION OF HARMONIC POWER FACTOR WITH FUNDAMENTAL SLIP.

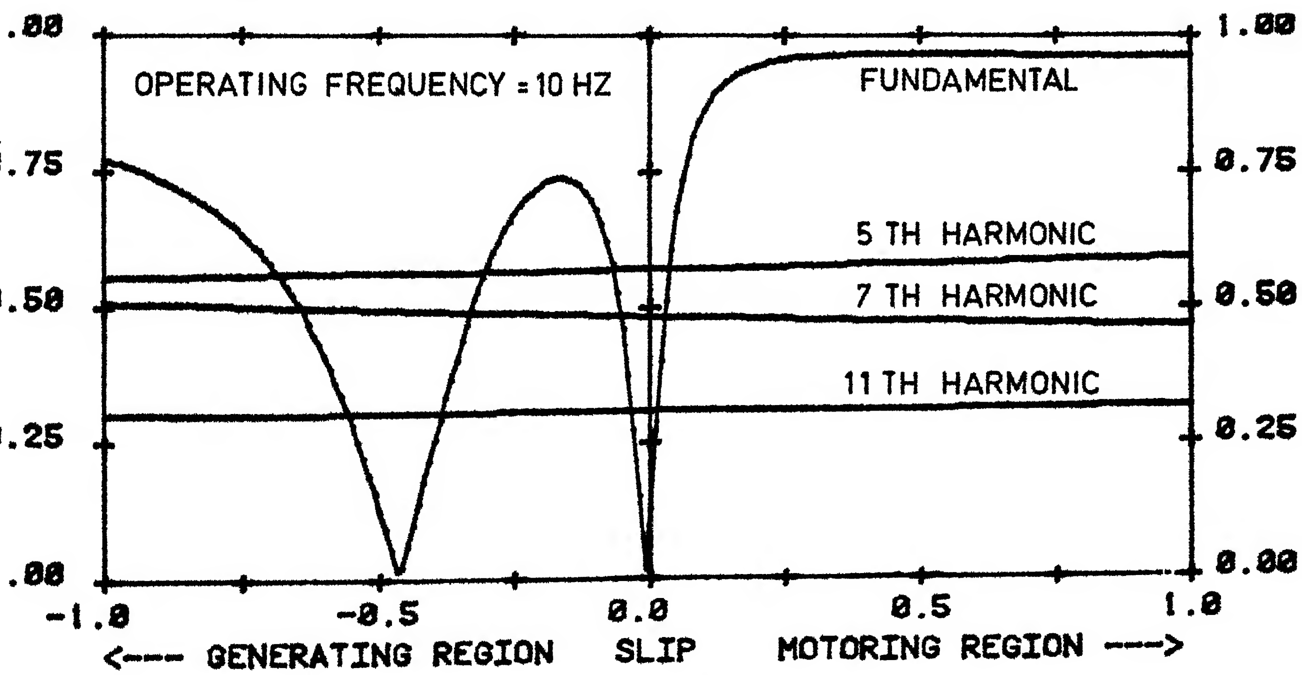


FIG 3.6(D) VARIATION OF HARMONIC POWER FACTOR WITH FUNDAMENTAL SLIP.

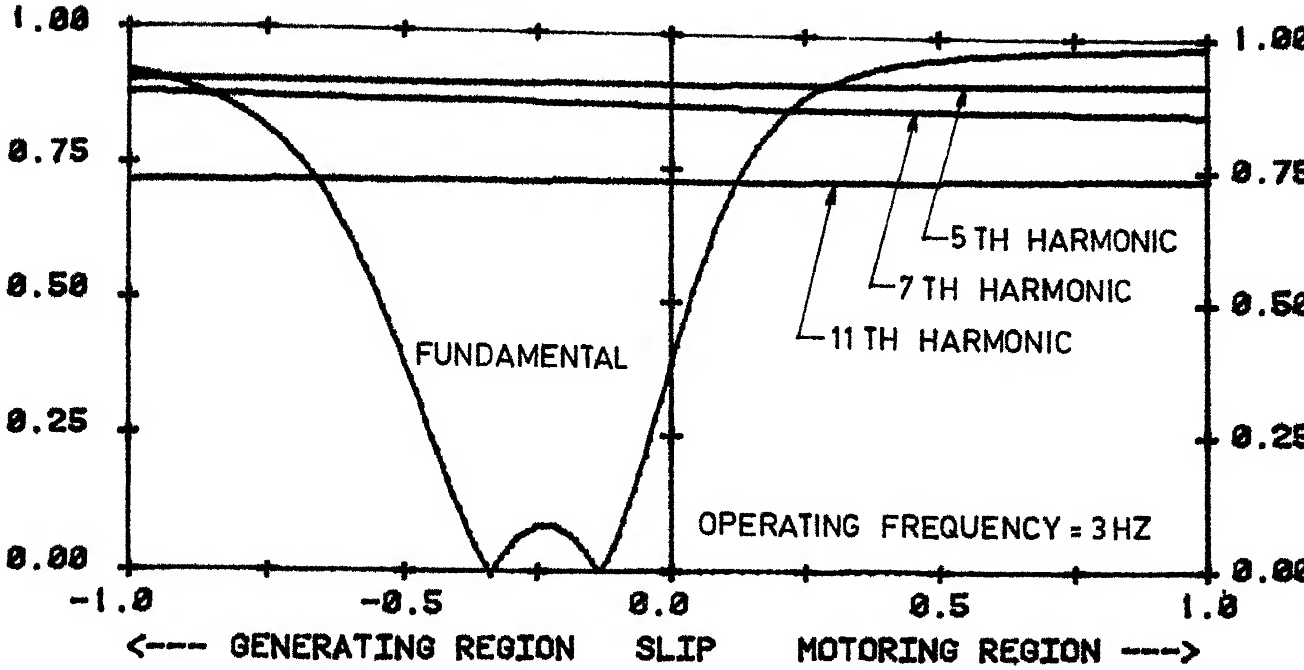


FIG 3.6(E) VARIATION OF HARMONIC POWER FACTOR WITH FUNDAMENTAL SLIP.

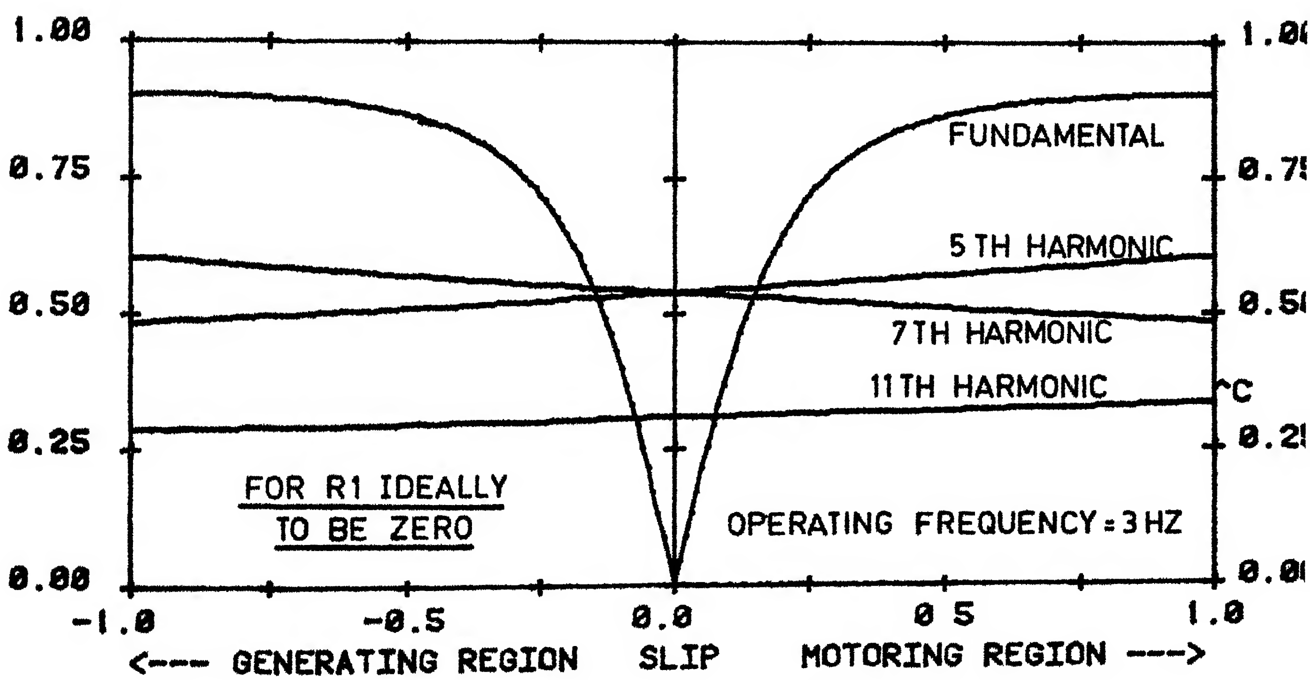


FIG 3.6(F) VARIATION OF HARMONIC POWER FACTOR WITH FUNDAMENTAL SLIP

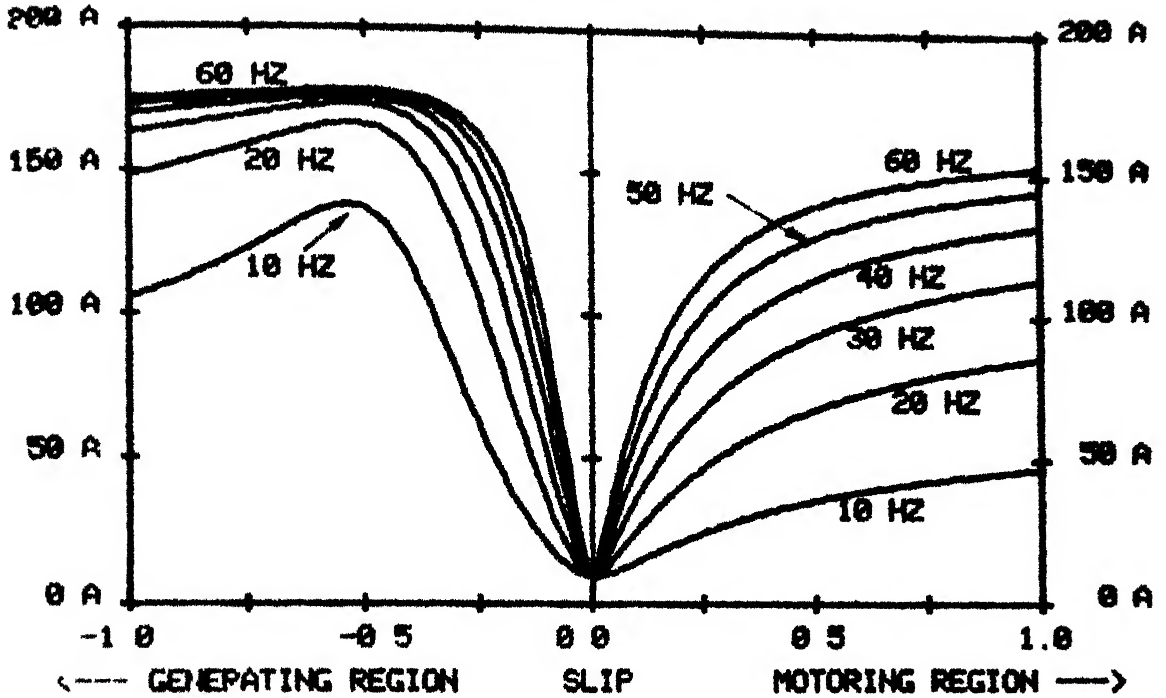


FIG 3.7(A) VARIATION OF STATOR FUNDAMENTAL R M.S CURRENT WITH SLIP

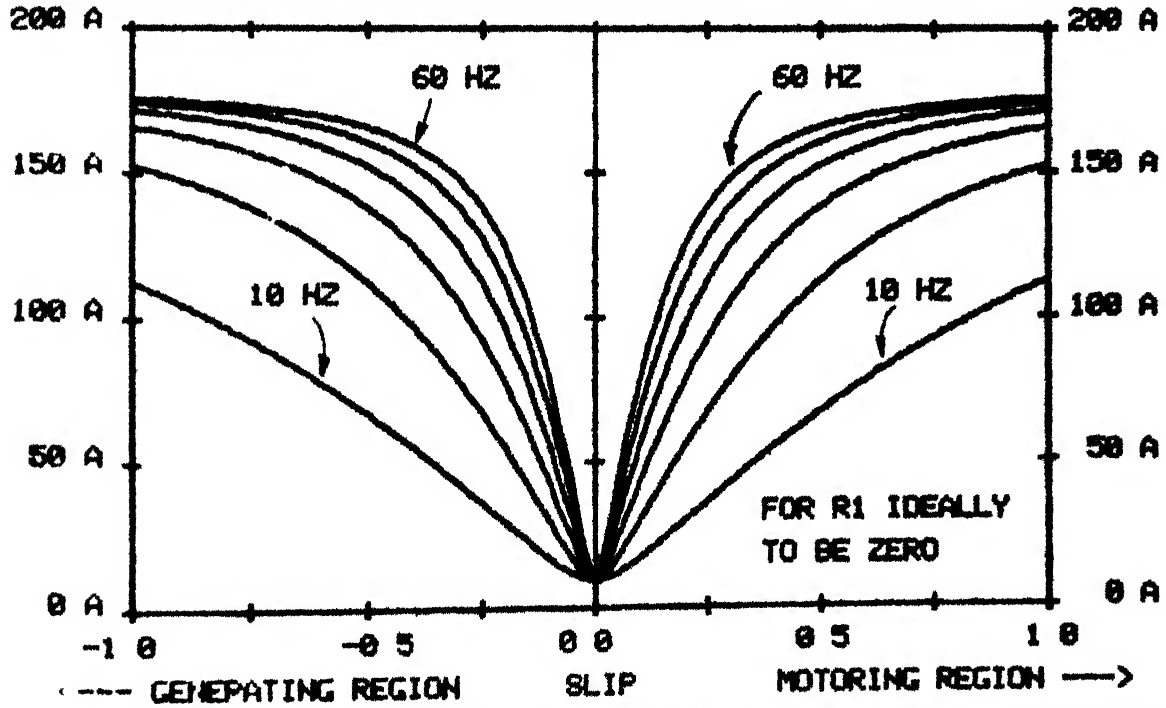
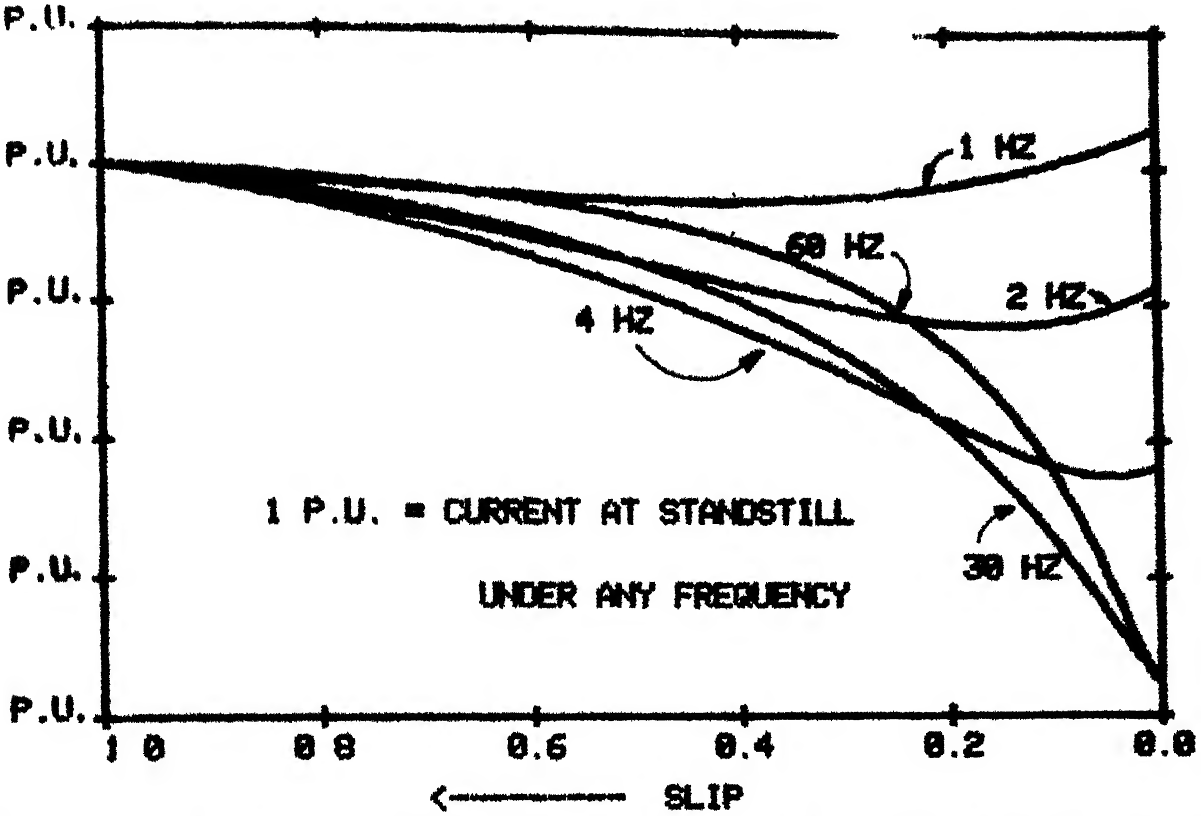
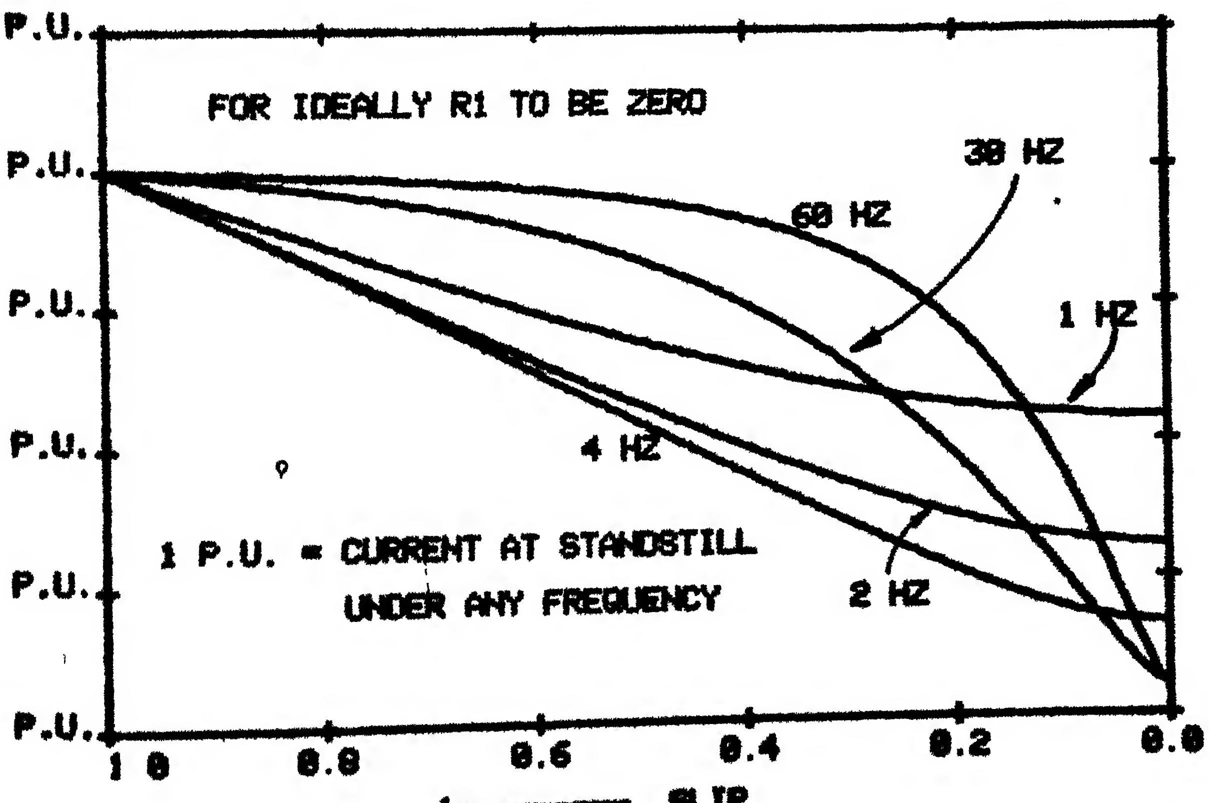


FIG 3 7(B) VARIATION OF STATOR FUNDAMENTAL R M S CURRENT WITH SLIP



3.7(C) VARIATION OF STATOR FUNDAMENTAL R.M.S. CURRENT WITH SLIP



3.7(D) VARIATION OF STATOR FUNDAMENTAL R.M.S. CURRENT WITH SLIP

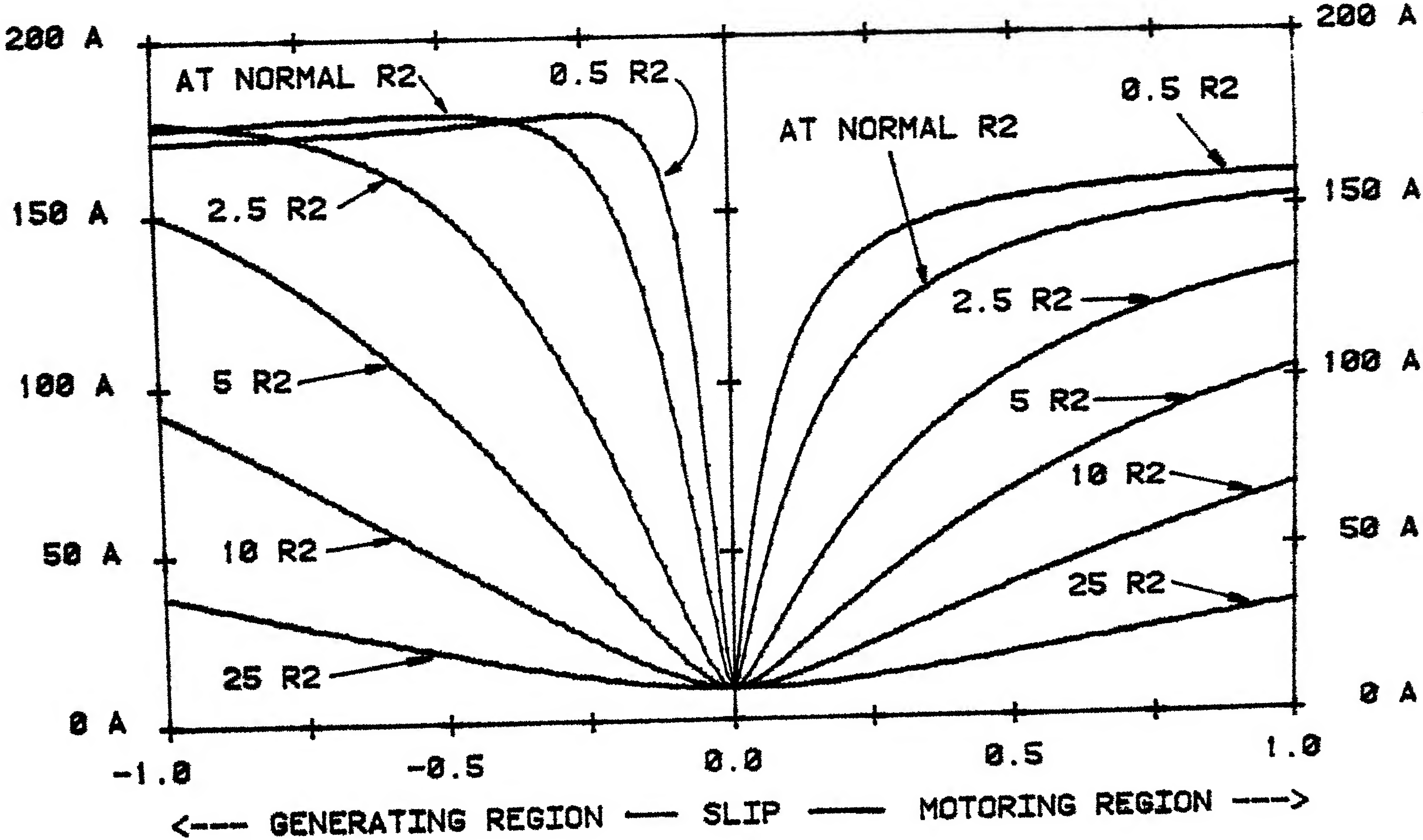


FIG 3.7(E) VARIATION OF FUNDAMENTAL STATOR R.M.S.CURRENT WITH SLIP FOR DIFFERENT ROTOR RESISTANCE.

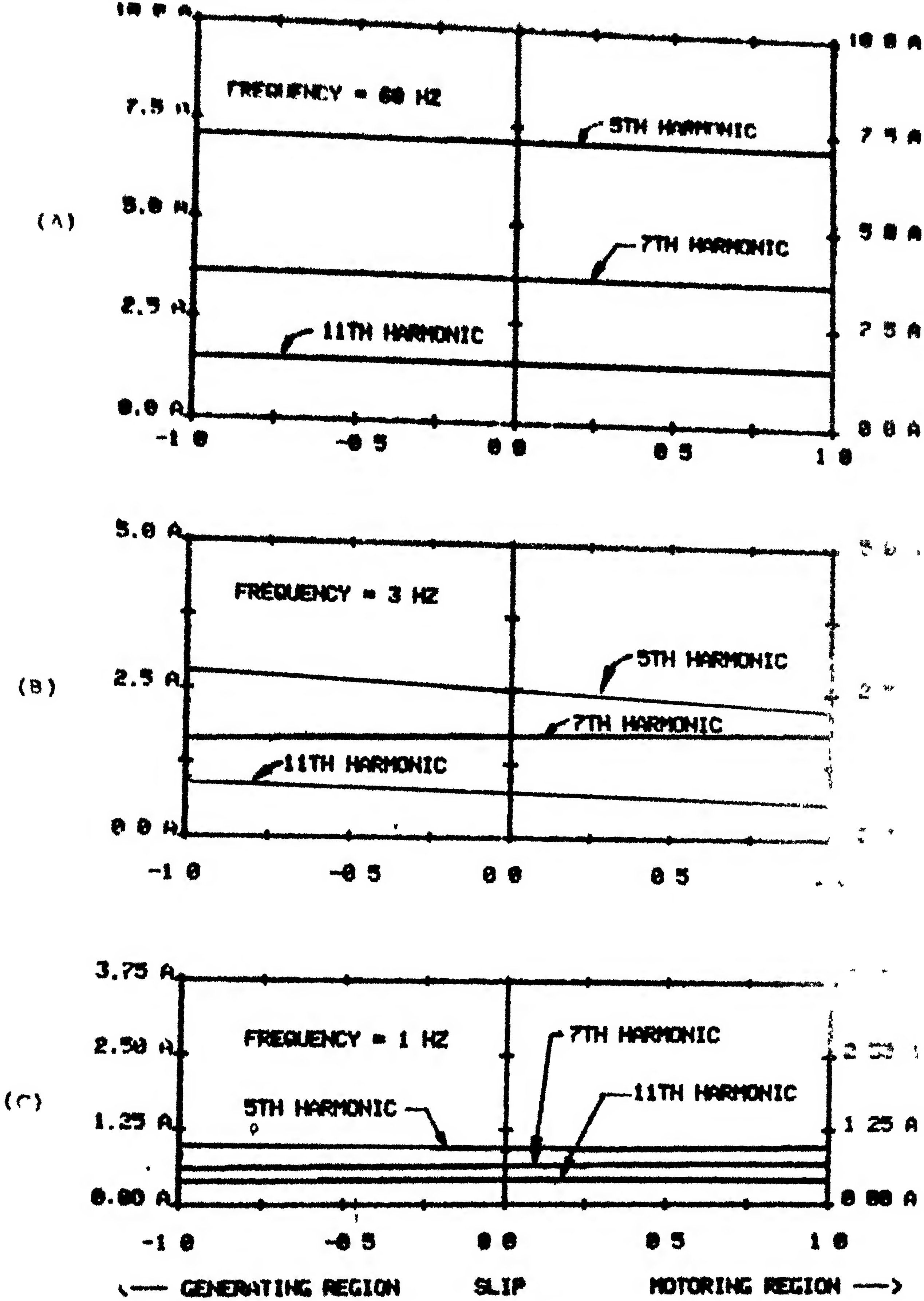


FIG 3 8 VARIATION OF R M S VALUE OF HARMONIC CURRENTS WITH SLIP

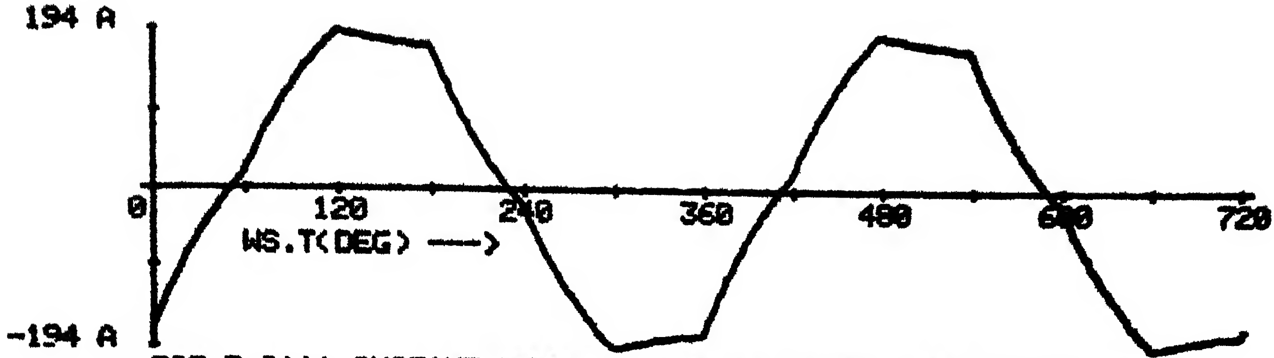


FIG 3.9(A) INSTANTANEOUS STATOR CURRENT AT SPEED 600 R.P.M.
OF FREQUENCY 60 HZ

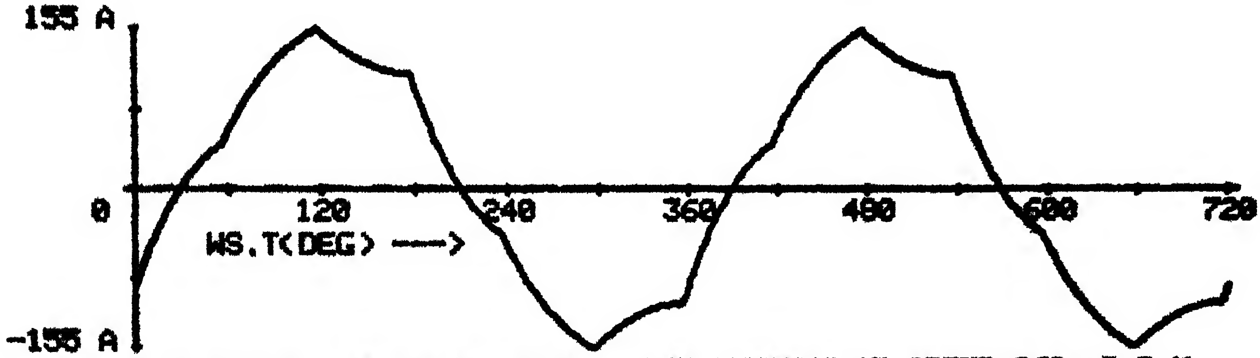


FIG 3.9(B) INSTANTANEOUS STATOR CURRENT AT SPEED 960 R.P.M.
OF FREQUENCY 60 HZ

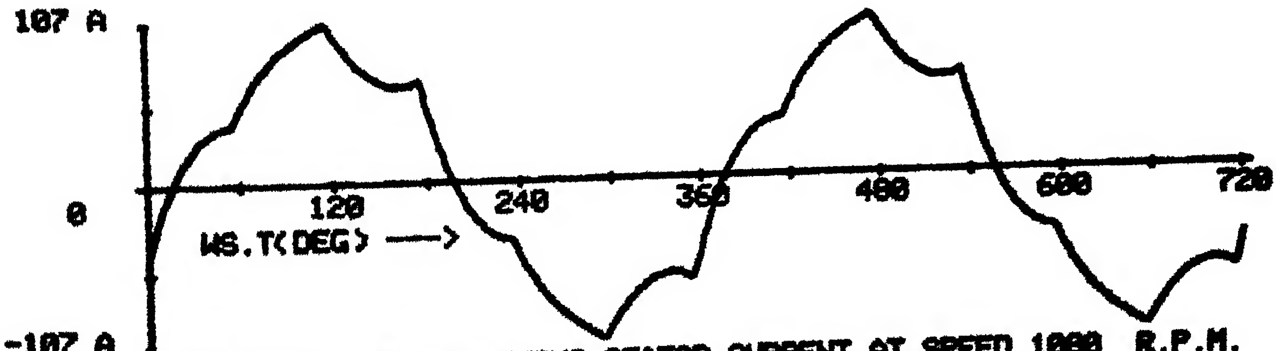


FIG 3.9(C) INSTANTANEOUS STATOR CURRENT AT SPEED 1000 R.P.M.
OF FREQUENCY 60 HZ

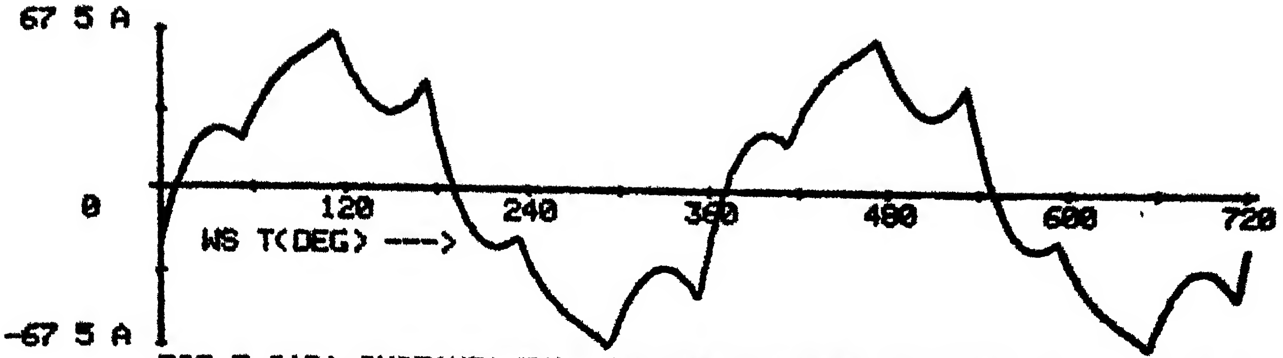


FIG 3 9(D) INSTANTANEOUS STATOR CURRENT AT SPEED 1140 R.P.M.
OF FREQUENCY 60 HZ

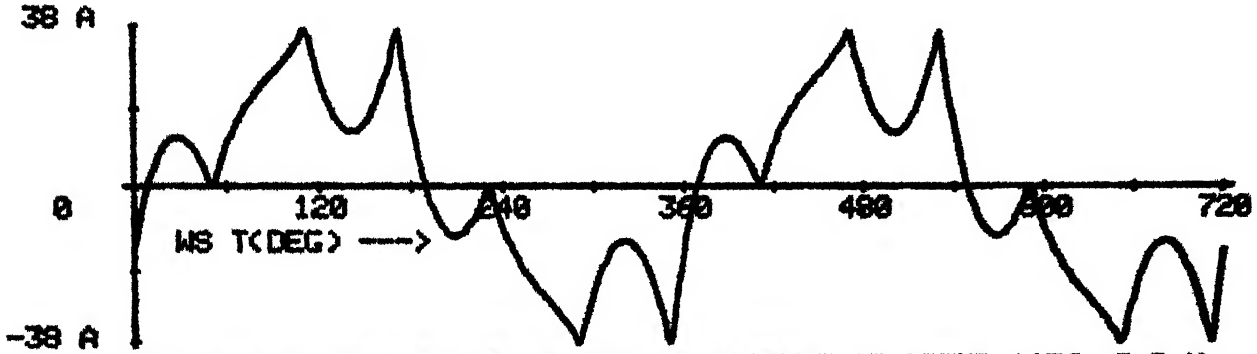


FIG 3.9(E) INSTANTANEOUS STATOR CURRENT AT SPEED 1176 R.P.M.
OF FREQUENCY 60 HZ

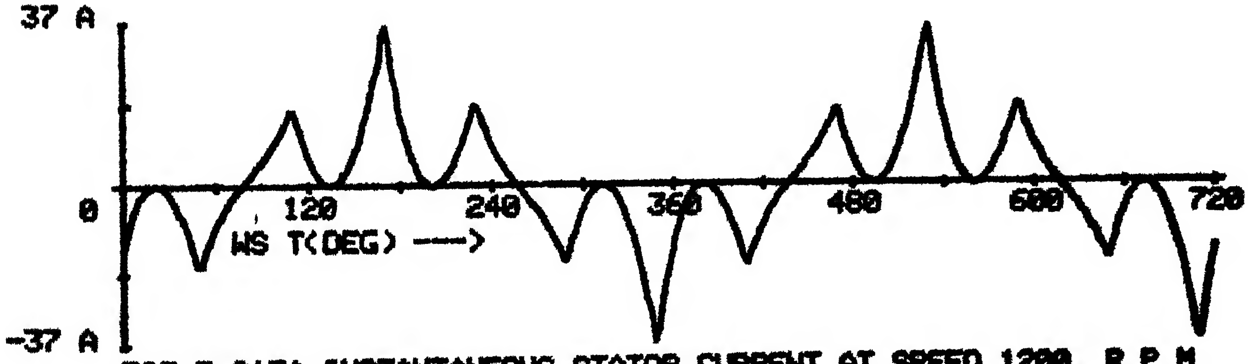


FIG 3.9(F) INSTANTANEOUS STATOR CURRENT AT SPEED 1200 R.P.M.
OF FREQUENCY 60 HZ

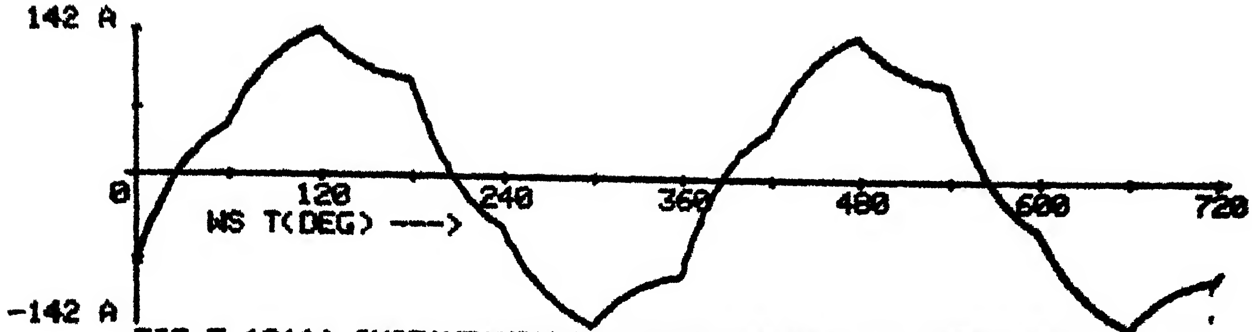


FIG 3 10(A) INSTANTANEOUS STATOR CURRENT AT SPEED 300 R P.M

OF FREQUENCY 30 HZ

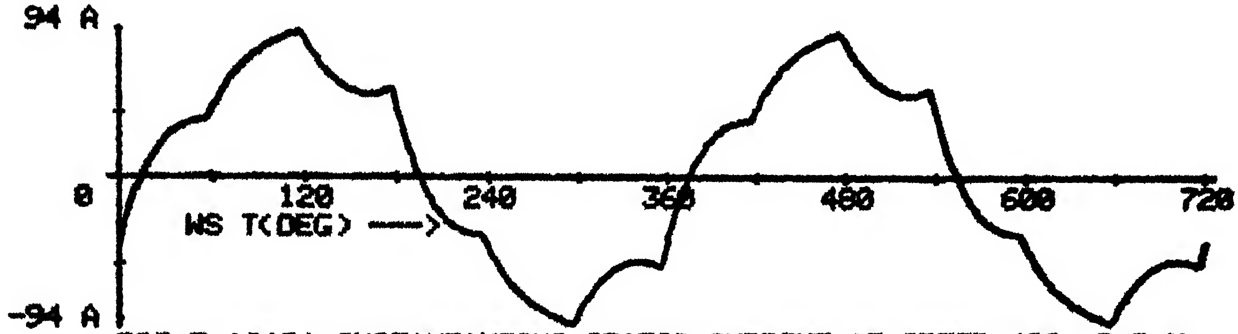


FIG 3.10(B) INSTANTANEOUS STATOR CURRENT AT SPEED 480 R.P.M.

OF FREQUENCY 30 HZ

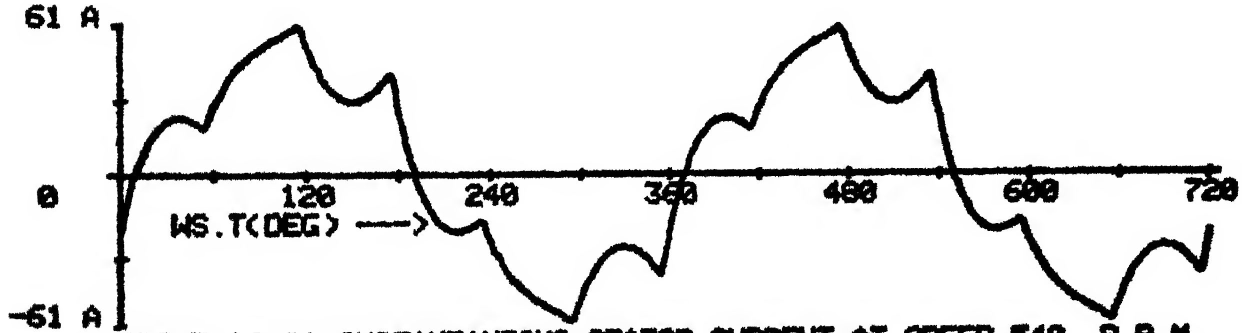


FIG 3.10(C) INSTANTANEOUS STATOR CURRENT AT SPEED 540 R.P.M.

OF FREQUENCY 30 HZ

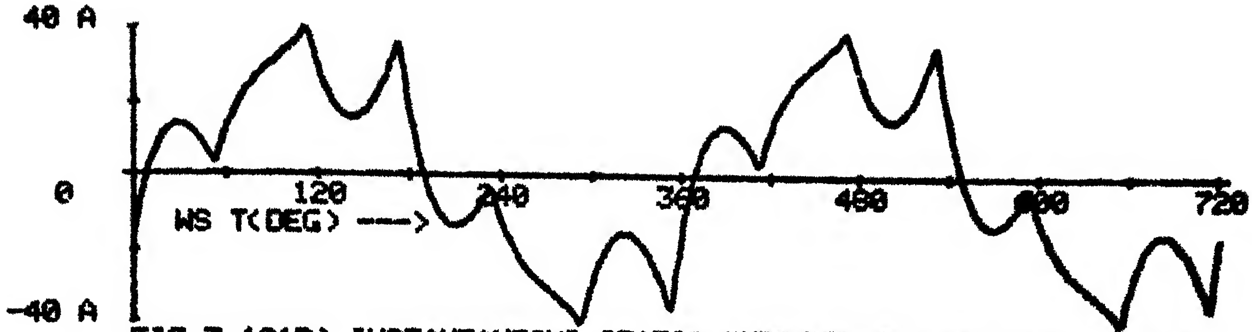


FIG 3 10(D) INSTANTANEOUS STATOR CURRENT AT SPEED 570 R.P.M.
OF FREQUENCY 30 HZ

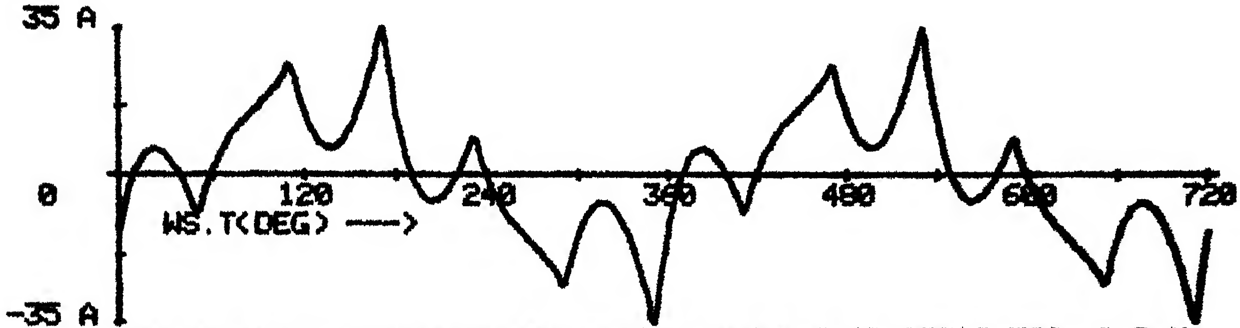


FIG 3 10(E) INSTANTANEOUS STATOR CURRENT AT SPEED 588 R.P.M.
OF FREQUENCY 30 HZ

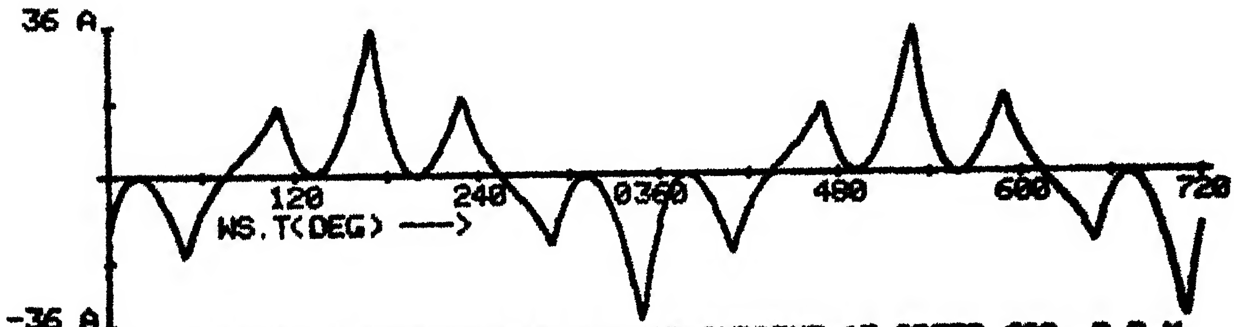


FIG 3 10(F) INSTANTANEOUS STATOR CURRENT AT SPEED 600 R.P.M.
OF FREQUENCY 30 HZ

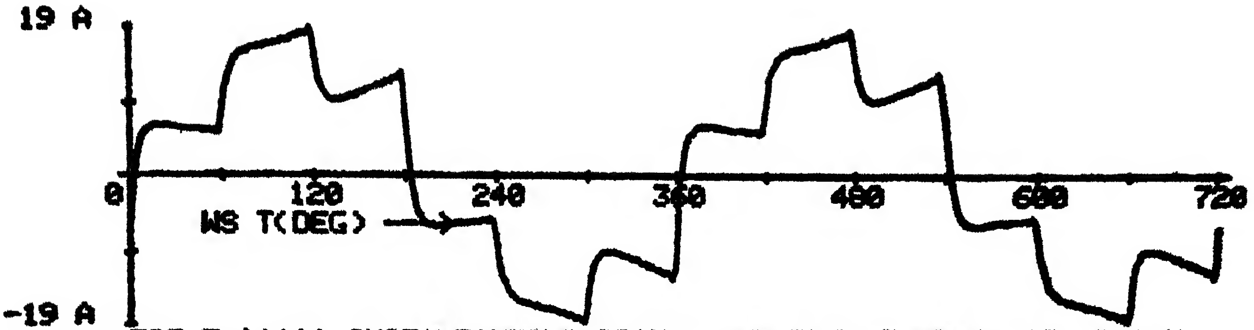


FIG 3 11(A) INSTANTANEOUS STATOR CURRENT AT SPEED 30 R.P.M.
OF FREQUENCY 3 HZ

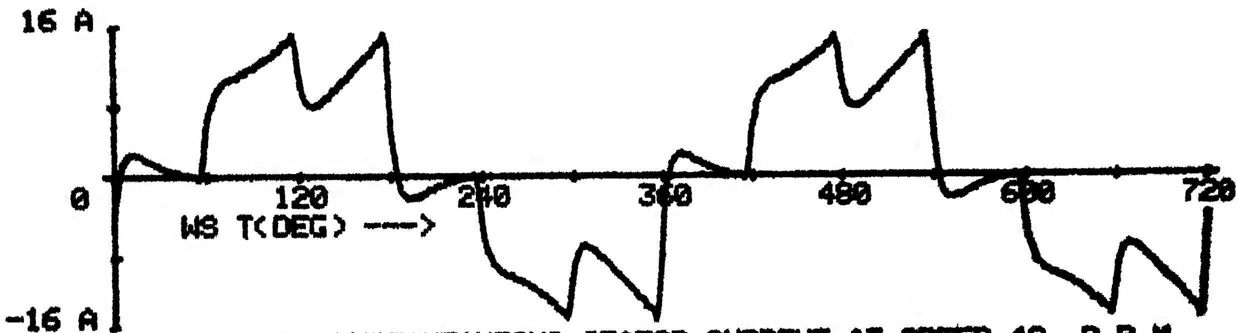


FIG 3 11(B) INSTANTANEOUS STATOR CURRENT AT SPEED 48 R.P.M
OF FREQUENCY 3 HZ

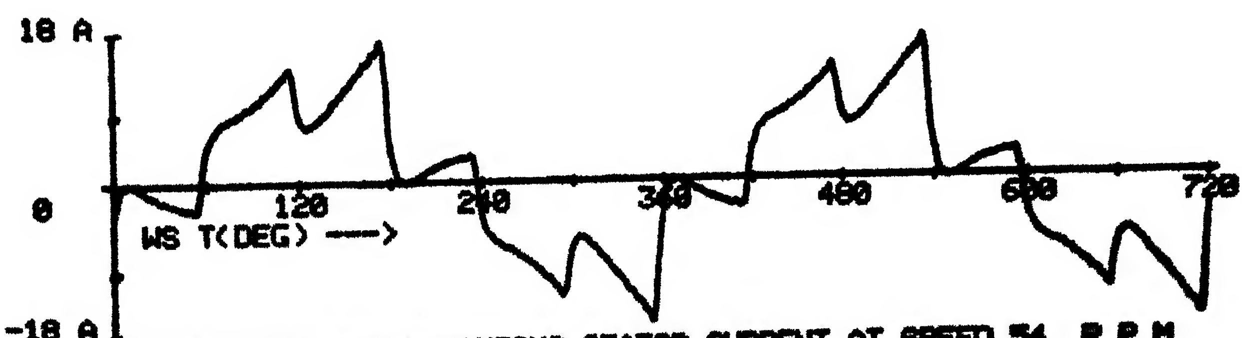


FIG 3 11(C) INSTANTANEOUS STATOR CURRENT AT SPEED 54 R.P.M.
OF FREQUENCY 3 HZ

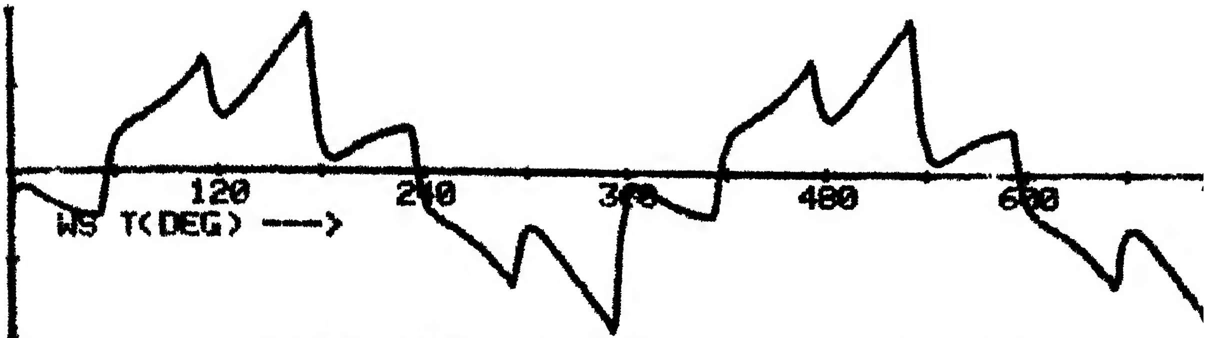


FIG 3 11(D) INSTANTANEOUS STATOR CURRENT AT SPEED 57 R P M

OF FREQUENCY 3 HZ

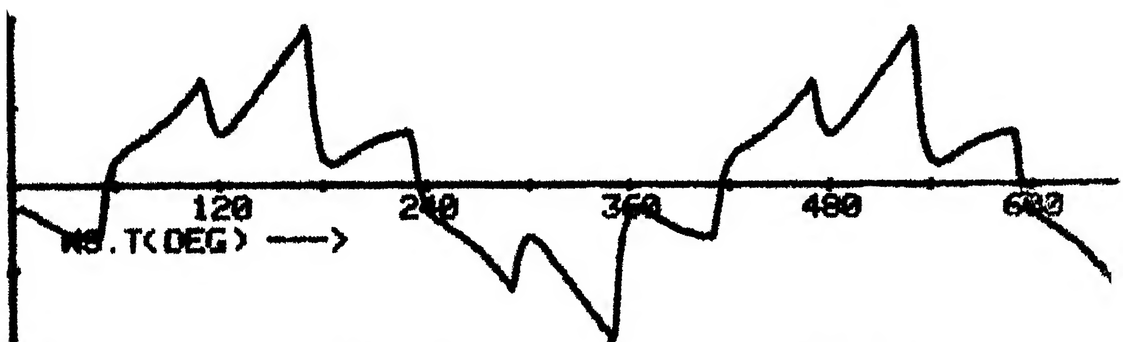


FIG 3.11(E) INSTANTANEOUS STATOR CURRENT AT SPEED 58 8

OF FREQUENCY 3 HZ

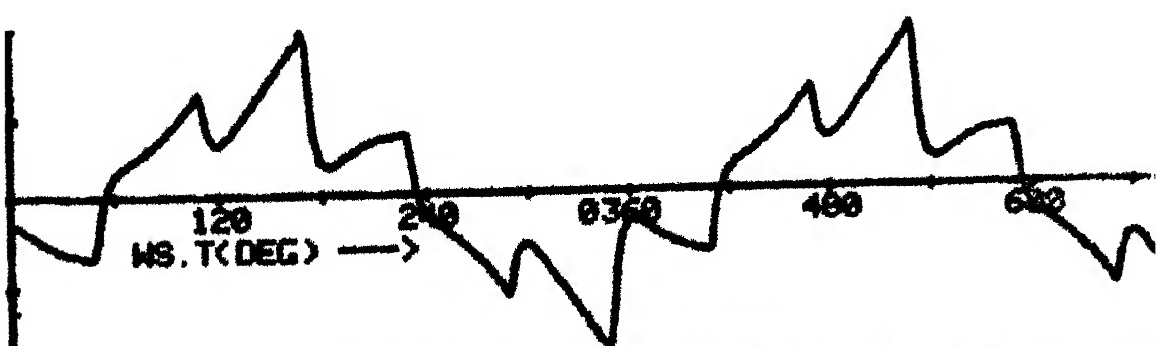


FIG 3.11(F) INSTANTANEOUS STATOR CURRENT AT SPEED 60 R P.M.

OF FREQUENCY 3 HZ

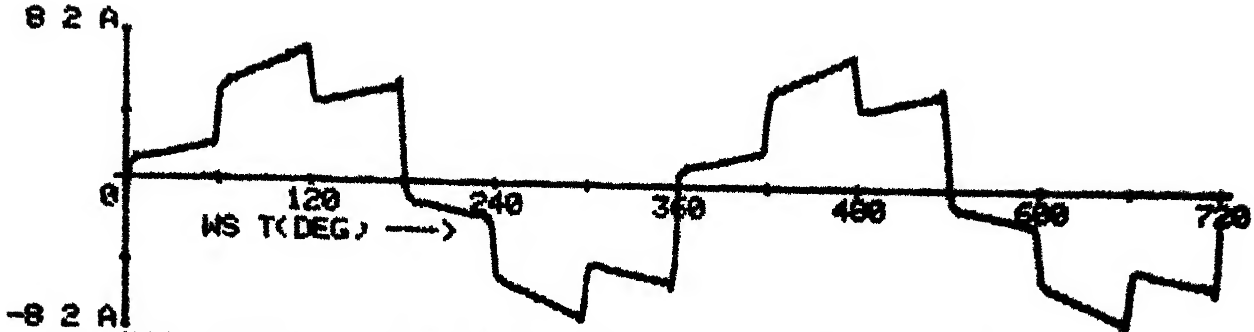


FIG 3.12(A) INSTANTANEOUS STATOR CURRENT AT SPEED 10 R.P.M.

OF FREQUENCY 1 HZ

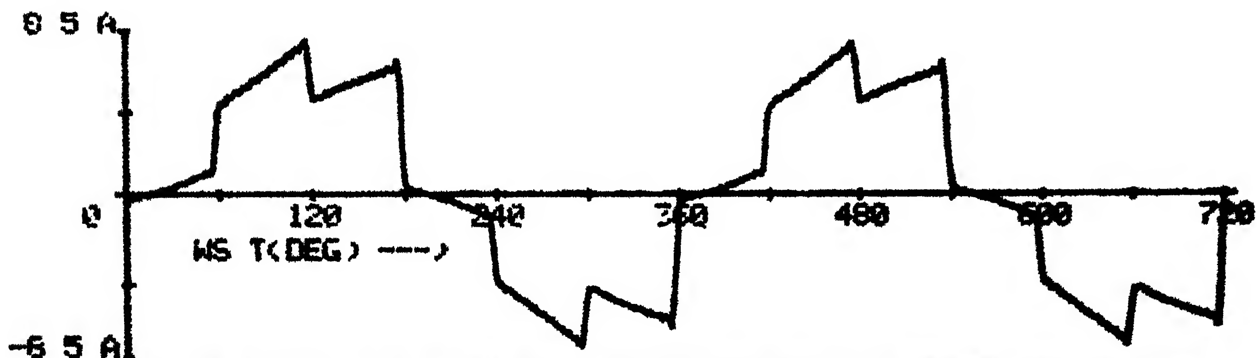


FIG 3 12(B INSTANTANEOUS STATOR CURRENT AT SPEED 16 R.P.M.

OF FREQUENCY 1 HZ

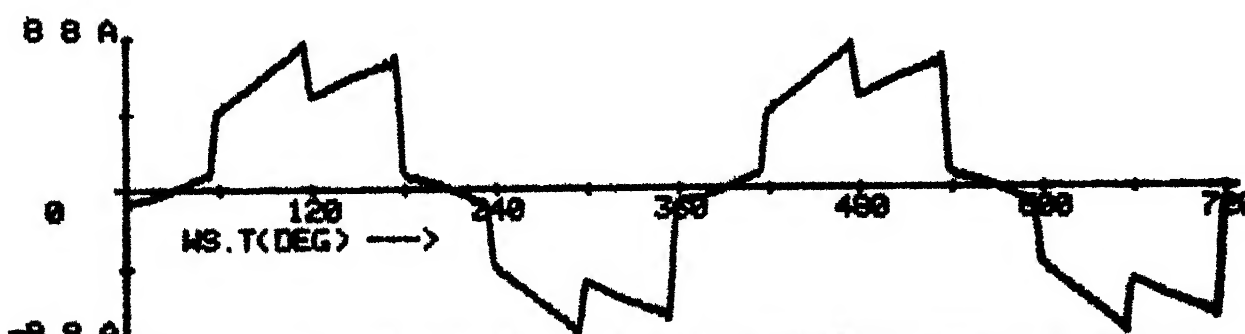


FIG 3.12(C) INSTANTANEOUS STATOR CURRENT AT SPEED 18 R.P.M.

OF FREQUENCY 1 HZ

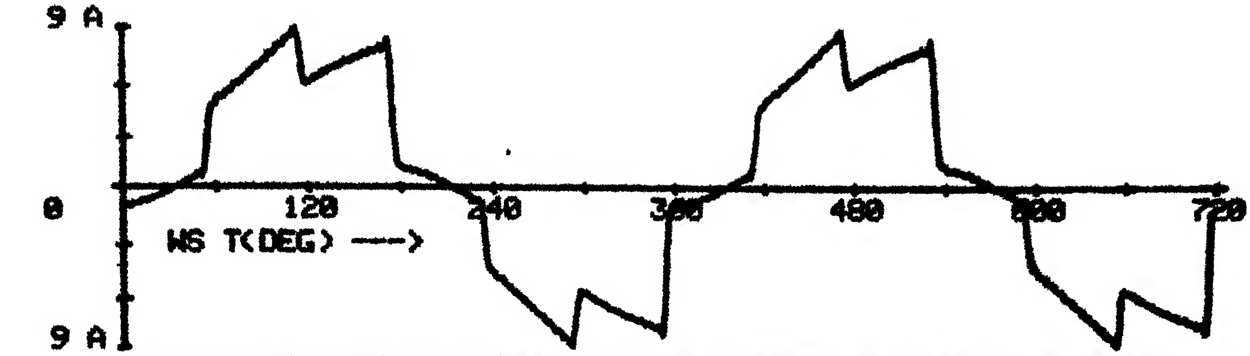


FIG 3.12(D) INSTANTANEOUS STATOR CURRENT AT SPEED 19 R.P M

OF FREQUENCY 1 HZ

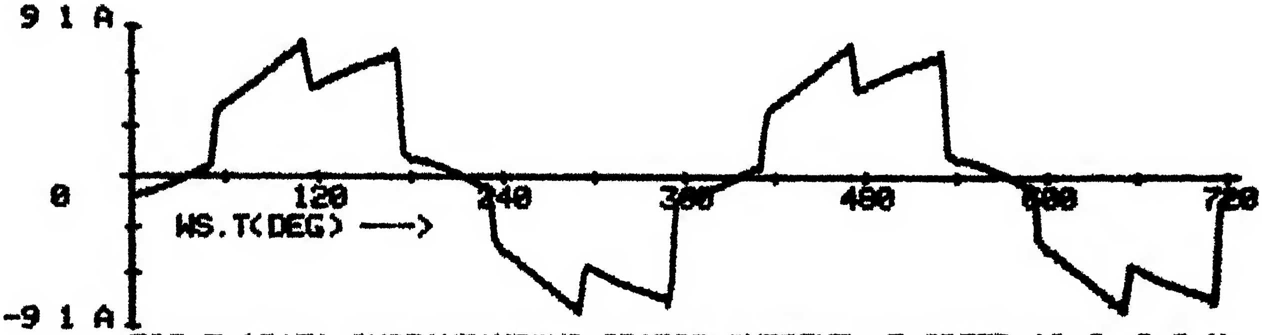


FIG 3 12(E) INSTANTANEOUS STATOR CURRENT AT SPEED 19.6 R.P.M.

OF FREQUENCY 1 HZ

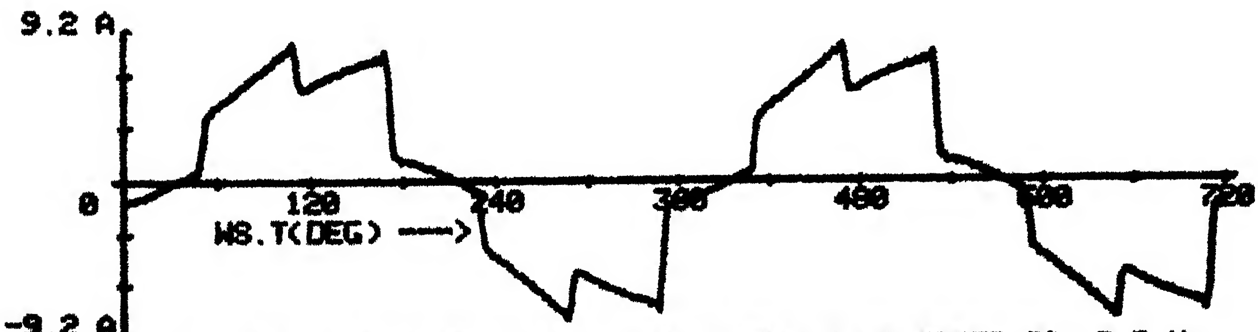


FIG 3.12(F) INSTANTANEOUS STATOR CURRENT AT SPEED 20 R.P.M

OF FREQUENCY 1 HZ

TABLE 3.1

VARIATION OF INDIVIDUAL CURRENT HARMONICS WITH FUNDAMENTAL SLIP FOR DIFFERENT INVERTER OPERATING FREQUENCY

Sl. No.	Fundamental slip (S)	Operating frequency 60 Hz				Operating frequency 30 Hz	
		K_{1q}	K_{5q}	K_{7q}	K_{11q}	K_{1q}	K_{5q}
1	-1.00	1.0000	0.0408	0.0208	0.0085	1.0000	0.0429
2	-0.90	1.0000	0.0406	0.0207	0.0084	1.0000	0.0423
3	-0.80	1.0000	0.0404	0.0207	0.0084	1.0000	0.0417
4	-0.70	1.0000	0.0403	0.0206	0.0084	1.0000	0.0410
5	-0.60	1.0000	0.0401	0.0205	0.0083	1.0000	0.0405
6	-0.50	1.0000	0.0401	0.0205	0.0083	1.0000	0.0404
7	-0.40	1.0000	0.0404	0.0206	0.0084	1.0000	0.0415
8	-0.30	1.0000	0.0418	0.0214	0.0087	1.0000	0.0465
9	-0.20	1.0000	0.0475	0.0243	0.0099	1.0000	0.0641
10	-0.10	1.0000	0.0775	0.0396	0.0161	1.0000	0.1343
11	0.00	1.0000	0.7711	0.3944	0.1602	1.0000	0.7567
12	0.10	1.0000	0.1054	0.0539	0.0219	1.0000	0.1923
13	0.20	1.0000	0.0695	0.0355	0.0144	1.0000	0.1179
14	0.30	1.0000	0.0588	0.0301	0.0122	1.0000	0.0934
15	0.40	1.0000	0.0540	0.0270	0.0112	1.0000	0.0815
16	0.50	1.0000	0.0513	0.0263	0.0107	1.0000	0.0746
17	0.60	1.0000	0.0497	0.0254	0.0103	1.0000	0.0702
18	0.70	1.0000	0.0485	0.0248	0.0101	1.0000	0.0670
19	0.80	1.0000	0.0477	0.0244	0.0099	1.0000	0.0647
20	0.90	1.0000	0.0471	0.0241	0.0098	1.0000	0.0630
21	1.00	1.0000	0.0460	0.0239	0.0097	1.0000	0.0616

(Note: All values are in p.u.).

Continued......

TABLE 3.1 (CONTINUED):

Sl. No.	Fundamental slip (S)	Operating frequency 30 Hz		Operating frequency 3 Hz			
		K_{7q}	K_{11q}	K_{1q}	K_{5q}	K_{7q}	K_{11q}
1	-1.00	0.0220	0.0090	1.0000	0.0798	0.0447	0.0276
2	-0.90	0.0217	0.0089	1.0000	0.0751	0.0425	0.0261
3	-0.80	0.0214	0.0088	1.0000	0.0703	0.0401	0.0244
4	-0.70	0.0211	0.0086	1.0000	0.0660	0.0380	0.0230
5	-0.60	0.0208	0.0085	1.0000	0.0638	0.0377	0.0223
6	-0.50	0.0207	0.0085	1.0000	0.0674	0.0395	0.0236
7	-0.40	0.0213	0.0087	1.0000	0.0824	0.0488	0.0289
8	-0.30	0.0239	0.0098	1.0000	0.1154	0.0689	0.0405
9	-0.20	0.0330	0.0135	1.0000	0.1730	0.1042	0.0609
10	-0.10	0.0602	0.0283	1.0000	0.2563	0.1558	0.0905
11	0.00	0.3897	0.1596	1.0000	0.3335	0.2045	0.1180
12	0.10	0.0991	0.0406	1.0000	0.3502	0.2166	0.1243
13	0.20	0.0608	0.0249	1.0000	0.3222	0.2070	0.1147
14	0.30	0.0482	0.0197	1.0000	0.2881	0.1813	0.1028
15	0.40	0.0421	0.0172	1.0000	0.2601	0.1652	0.0931
16	0.50	0.0385	0.0158	1.0000	0.2387	0.1529	0.0857
17	0.60	0.0362	0.0148	1.0000	0.2223	0.1437	0.0801
18	0.70	0.0346	0.0142	1.0000	0.2095	0.1366	0.0758
19	0.80	0.0335	0.0137	1.0000	0.1992	0.1301	0.0723
20	0.90	0.0326	0.0133	1.0000	0.1908	0.1267	0.0695
21	1.00	0.0319	0.0130	1.0000	0.1838	0.1231	0.0672

(Note: All values are in p.u.).

TABLE 3.2

VARIATION OF INDIVIDUAL HARMONIC POWER FACTORS WITH FUNDAMENTAL SLIP FOR DIFFERENT INVERTER OPERATING FREQUENCY

Sl. No.	Fundamental slip (S)	Operating frequency 60 Hz				Operating frequency 30 Hz	
		Cos ϕ_{1q}	Cos ϕ_{5q}	Cos ϕ_{7q}	Cos ϕ_{11q}	Cos ϕ_{1q}	Cos ϕ_{5q}
1	-1.0	0.2125	0.1104	0.0982	0.0528	0.3968	0.2169
2	-0.9	0.1919	0.1108	0.0974	0.0529	0.3619	0.2176
3	-0.8	0.1658	0.1112	0.0967	0.0530	0.3162	0.2184
4	-0.7	0.1318	0.1116	0.0960	0.0531	0.2544	0.2192
5	-0.6	0.0859	0.1121	0.0953	0.0533	0.1677	0.2200
6	-0.5	0.0210	0.1125	0.0946	0.0534	0.0420	0.2209
7	-0.4	0.0759	0.1130	0.0940	0.0535	0.1430	0.2217
8	-0.3	0.2290	0.1135	0.0934	0.0536	0.3996	0.2226
9	-0.2	0.4742	0.1140	0.0928	0.0538	0.6789	0.2236
10	-0.1	0.7852	0.1145	0.0922	0.0539	0.8518	0.2245
11	0.0	0.0214	0.1150	0.0917	0.0540	0.0427	0.2255
12	0.1	0.8902	0.1155	0.0911	0.0542	0.9307	0.2266
13	0.2	0.7986	0.1161	0.0906	0.0543	0.9169	0.2276
14	0.3	0.7228	0.1167	0.0901	0.0545	0.8902	0.2287
15	0.4	0.6669	0.1173	0.0897	0.0546	0.8646	0.2299
16	0.5	0.6255	0.1179	0.0892	0.0547	0.3424	0.2310
17	0.6	0.5941	0.1186	0.0887	0.0549	0.8234	0.2322
18	0.7	0.5696	0.1192	0.0883	0.0550	0.8073	0.2335
19	0.8	0.5501	0.1199	0.0879	0.0552	0.7936	0.2348
20	0.9	0.5342	0.1207	0.0875	0.0554	0.7818	0.2362
21	1.0	0.5210	0.1214	0.0871	0.0555	0.7716	0.2376

Continued.....

TABLE 3.2 (CONTINUED):

Sl. No.	Fundamental slip (S)	Operating frequency 30 Hz		Operating frequency 3 Hz			
		Cos ϕ_{7q}	Cos ϕ_{11q}	Cos ϕ_{1q}	Cos ϕ_{5q}	Cos ϕ_{7q}	Cos ϕ_{11q}
1	-1.0	0.1935	0.1051	0.9289	0.9092	0.8856	0.7247
2	-0.9	0.1920	0.1054	0.9023	0.9097	0.8844	0.7255
3	-0.8	0.1906	0.1056	0.8567	0.9102	0.8831	0.7262
4	-0.7	0.1893	0.1058	0.7746	0.9107	0.8819	0.7270
5	-0.6	0.1880	0.1061	0.6265	0.9112	0.8807	0.7277
6	-0.5	0.1867	0.1063	0.3918	0.9118	0.8796	0.7285
7	-0.4	0.1855	0.1066	0.1249	0.9123	0.8784	0.7293
8	-0.3	0.1843	0.1068	0.0560	0.9129	0.8773	0.7301
9	-0.2	0.1832	0.1071	0.0815	0.9134	0.8762	0.7309
10	-0.1	0.1821	0.1073	0.0766	0.9140	0.8751	0.7317
11	0.0	0.1811	0.1076	0.3931	0.9146	0.8740	0.7325
12	0.1	0.1800	0.1079	0.6901	0.9152	0.8729	0.7333
13	0.2	0.1790	0.1081	0.8502	0.9158	0.8719	0.7342
14	0.3	0.1781	0.1084	0.9207	0.9164	0.8709	0.7350
15	0.4	0.1772	0.1087	0.9526	0.9171	0.8699	0.7359
16	0.5	0.1763	0.1090	0.9684	0.9177	0.8689	0.7367
17	0.6	0.1754	0.1093	0.9771	0.9183	0.8679	0.7376
18	0.7	0.1745	0.1096	0.9822	0.9190	0.8670	0.7385
19	0.8	0.1737	0.1099	0.9854	0.9197	0.8660	0.7394
20	0.9	0.1729	0.1102	0.9875	0.9204	0.8651	0.7403
21	1.0	0.1721	0.1105	0.9890	0.9211	0.8642	0.7413

TABLE 3.3

VARIATION OF SLIP DUE TO POSITIVE AND NEGATIVE SEQUENCE HARMONICS WITH FUNDAMENTAL MOTOR SLIP

Sl. No.	Fundamental slip (S)	Negative sequence 5th harmonic slip (s_5)	Positive sequence 7th harmonic slip (s_7)	Negative sequence 11th harmonic slip (s_{11})	Positive sequence 13th harmonic slip (s_{13})	Negative sequence 17th harmonic slip (s_{17})	Positive sequence 19th harmonic slip (s_{19})
1	1.00	1.00	1.0000	1.0000	1.0000	1.0000	1.0000
2	0.90	1.02	0.9857	1.0091	0.9923	1.0058	0.9947
3	0.80	1.04	0.9714	1.0182	0.9846	1.0117	0.9894
4	0.70	1.06	0.9571	1.0273	0.9769	1.0176	0.9842
5	0.60	1.08	0.9428	1.0364	0.9692	1.0235	0.9789
6	0.50	1.10	0.9285	1.0455	0.9615	1.0294	0.9736
7	0.40	1.12	0.9142	1.0546	0.9538	1.0353	0.9684
8	0.30	1.14	0.9000	1.0637	0.9461	1.0412	0.9631
9	0.20	1.16	0.8857	1.0728	0.9384	1.0470	0.9578
10	0.10	1.18	0.8714	1.0819	0.9307	1.0529	0.9526
11	0.00	1.20	0.8571	1.0910	0.9230	1.0588	0.9473
12	-0.10	1.22	0.8428	1.1001	0.9153	1.0647	0.9421
13	-0.20	1.24	0.8285	1.1091	0.9076	1.0706	0.9368
14	-0.30	1.26	0.8142	1.1152	0.9000	1.0765	0.9315
15	-0.40	1.28	0.8000	1.1273	0.8923	1.0823	0.9263
16	-0.50	1.30	0.7857	1.1364	0.8846	1.0882	0.9210
17	-0.60	1.32	0.7714	1.1455	0.8769	1.0941	0.9157
18	-0.70	1.34	0.7571	1.1546	0.8692	1.1000	0.9105
19	-0.80	1.36	0.7428	1.1637	0.8615	1.1058	0.9052
20	-0.90	1.38	0.7285	1.1728	0.8538	1.1117	0.9000
21	-1.00	1.40	0.7142	1.1819	0.846	1.1176	0.894

Chapter - 4

COMPUTATION OF THE ELECTROMAGNETIC TORQUE IN SIX STEP V.S.I. FED INDUCTION MOTOR

4.1 INTRODUCTION

The thyristor inverter speed control system of a three phase induction motor is used widely in industry. However, the thyristor inverter generates a distorted wave voltage and produces a ripple torque. Generally fundamental of the voltage produces the average output torque and the harmonics produce increased losses and torque fluctuations. For this reason it is very important to analyze the instantaneous torque. Earlier methods for analysing instantaneous torque do not always clarify analytically the relationship between machine constants and ripple torque. Methods have also been proposed to solve voltage equations analytically or by Fourier series expansion. However, they require complicated calculations and the obtained results are complicated.

The model normally used for studying the steady state behaviour of the machine is the single phase equivalent circuit [Figure 3.2]. Klingshirn and Jordon [14] have detailed the method of calculating the currents, losses and other performance characteristics under steady state, using the single phase equivalent circuit and balanced nonsinusoidal source. Torque fluctuations have not been obtained by them using the equivalent circuit. A method of using the aforesaid equivalent circuit

model of the machine for evaluating the torque fluctmations with nonsinusoidal applied voltage has been detailed in this chapter. In this study, the equations describing system operation have been linearised by assuming a constant rotor speed as well as a constant inverter (d.c.) voltage. The constant speed constraint is required in these calculations so that superposition could be employed.

In section 4.2, expression of electromagnetic torque has been derived with the help of machine equations.

Nature of electromagnetic torque due to ideal six stepped inverter output voltage has been discussed in section 4.3. Expression for the instantaneous electromagnetic torque has been derived in section 4.3.1, considering all existing harmonics of the six stepped voltage waveform. It has been shown that the above torque consists of steady and pulsating harmonic torque. Steady harmonic or average torque characteristics has bee discussed in section 4.3.2 for different operating frequencies and machine parameters. Pulsating harmonic torques have been discussed in section 4.3.3 in the same fashion Since contribution due to high order pulsating harmonic torques are very less as compared to lowest sixth harmonic torque, only pulsating sixth harmonic torque is considered. Instantaneous pulsating harmonic torques are also plotted on computer for different frequency and speed of the machine.

Lastly, in section 4.4 effect of harmonic torques in

$$0 = \frac{3}{2} r_2' (i_{2q}'^2 + i_{2d}'^2) + \frac{3}{2} (i_{2q}' . p\lambda_{2q}' + i_{2d}' . p\lambda_{2d}')$$

$$+ \frac{3}{2} \{ - \lambda_{2q}' (p\theta_s) . i_{2d}' + \lambda_{2d}' (p\theta_s) . i_{2q}' \} \quad (4.5)$$

Again adding equation (4.3) and equation (4.5)

$$p_1 = [\frac{3}{2} r_1 (i_{1q}^2 + i_{1d}^2)] + [\frac{3}{2} r_2' (i_{2q}' + i_{2d}'^2)]$$

$$+ [\frac{3}{2} (i_{1q} . p\lambda_{1q} + i_{1d} . p\lambda_{1d}) + \frac{3}{2} (i_{2q}' . p\lambda_{2q}' + i_{2d}' . p\lambda_{2d}')]$$

$$+ [\frac{3}{2} \omega_e (-\lambda_{1q} . i_{1d} + \lambda_{1d} . i_{1q}) + \{ \frac{3}{2} - \lambda_{2q}' (p\theta_s) . i_{2d}'$$

$$+ \lambda_{2d}' (p\theta_s) . i_{2q}' \}] \quad (4.6)$$

i.e. $p_1 = [p_s] + [p_r] + [p_1] + [p_m]$ (4.7)

where, p_s : stator copper loss

p_r : rotor copper loss

p_1 : instantaneous power across stator and rotor self and stator rotor mutual inductance,

and p_m : internal mechanical power which is as follows:

$$p_m = \frac{3}{2} \omega_e (\lambda_{1q} . i_{1d} + \lambda_{1d} . i_{1q}) + \{ \frac{3}{2} - \lambda_{2q}' (p\theta_s) i_{2d}'$$

$$+ \lambda_{2d}' (p\theta_s) i_{2q}' \} \quad (4.8)$$

For a motor revolving at constant speed, in the steady state the above equation can be written with the help of equations (2.9a), (2.9b), (2.10a) and (2.10b) as follows:

$$P_m = \frac{3}{2} M.\omega_e (1-S) (i_{1q}.i'_{2d} - i_{1d}.i'_{2q}) \quad (4.9)$$

where rotor is revolving at a slip 'S'

and the Q axis is advancing continuously with respect to a point on the rotor at the rate of $d\theta_s/dt = p\theta_s = S.\omega_e$.

Therefore, P_g (power across the air gap)

$$= \frac{3}{2} M.\omega_e (i_{1q}.i'_{2d} - i_{1d}.i'_{2q}) \quad (4.10)$$

and electromagnetic torque can be obtained as

$$t_e = \frac{3}{2} M. \frac{P}{2} (i_{1q}\, i'_{2d} - i_{1d}.i'_{2q}) \quad (4.11)$$

4.3 NATURE OF TORQUE IN VOLTAGE SOURCE INVERTER FED INDUCTION MOTOR

In this section the nature of harmonic components in the torque produced by the induction motor has been established. Nonsinusoidal input voltage fed to the motor can be analysed into fundamental component and a series of harmonics. Neglecting magnetic saturation, the motor may be regarded as linear device and the principle of superposition can be applied. The resultant effect to the above-mentioned supply is then obtained as a summation of the responses to the individual input components. The fundamental component above produces the useful motoring torque and the nontriplen odd positive and negative sequence harmonics of the six step V.S.I.,yield pulsating harmonic torques having harmonic frequencies which are multiple of six times the inverter operating frequency. Therefore, it is very

important to analyse the instantaneous torque. Though in general applications where variable voltage as well as varying frequency are obtainable by inverters, the designers normally try to establish such an inverter output voltage which is more or less close to sinusoidal by means of some modulation strategy.

4.3.1 Instantaneous Electromagnetic Torque in Voltage Source Inverter Fed Induction Motor

Considering the fundamental expression of instantaneous electromagnetic torque [Equation 4.11], the expression for electromagnetic torque in six step voltage source inverter fed induction motor is as follows:

$$t_e = M_c \left[(\Sigma i_{1nq})(\Sigma i'_{2nd}) - (\Sigma i_{1nd})(\Sigma i'_{2nq}) \right] \quad (4.12)$$

where n = 1,5,7,11,13.......∞ and $M_c = (1.5)\, M\, P/2$.

Now disuniting the expression in the above equation,

$$t_e = \left[M_c \sum_{n=1,5,7}^{\infty} A_{nn} \right] + \left[M_c \sum_{\substack{m=1 \\ n=(6-m)}}^{m=5} A_{mn} \right.$$

$$+ \left. M_c \sum_{\substack{m=1,5,7 \\ n=(6+m)}}^{\infty} (A_{mn} + A_{nm}) \right] + \left[M_c \sum_{\substack{m=1,5 \\ n=(12-m)}}^{m=11} A_{mn} \right.$$

$$+ \left. M_c \sum_{\substack{m=1,5,7 \\ n=(12+m)}}^{\infty} (A_{mn} + A_{nm}) \right] + [\ ] + [\ ] + \dots \quad (4.13)$$

where $A_{mn} = (i_{1mq} \cdot i'_{2nd} - i_{1nd} \cdot i'_{2mq})$

$A_{nm} = (i_{1nq} \cdot i'_{2md} - i_{1md} \cdot i'_{2nq})$

and $A_{nn} = (i_{1nq} \cdot i'_{2nd} - i_{1nd} \cdot i'_{2nq})$

$$\text{or } t_e = [M_c \sum_{n=1,5,7}^{\infty} A_{nn}] + [\sum_{h=6,12,18}^{\infty} \{ M_c \sum_{\substack{m=1,5,\\ n=(h-m)}}^{(h-1)} A_{mn}$$

$$+ M_c \sum_{\substack{m=1,5,7\\ n=(h+m)}}^{\infty} (A_{mn} + A_{nm}) \}] \qquad (4.14)$$

or $t_e = [t_{sh}] + [t_{ph}]$

where t_{sh} : steady harmonic torque

and t_{ph} : pulsating harmonic torque.

Therefore, $t_{sh} = t_{s1}$ (due to fundamental component of stator and rotor currents) + t_{s5} (due to fifth harmonic component of stator and rotor currents) + t_{s7} (due to seventh harmonic component of stator and rotor currents) + ...∞ (4.15)

and $t_{ph} = t_{p6} + t_{p12} + t_{p18} + t_{p24} + \ldots\ldots\infty$ (4.16)

where $t_{p6} = t_{p6(1,5)}$ {due to fundamental and fifth harmonic component of stator and rotor currents } + $t_{p6(1,7)}$ {due to fundamental and seventh harmonic component of stator and rotor currents }+ ....∞

$t_{p12} = t_{p12(1,11)}$ {due to fundamental and eleventh harmonic component of stator and rotor currents }+ $t_{p12(5,7)}$ {due to fifth

and seventh harmonic component of stator and motor currents }

+ ....∞

and so on.

Therefore, as shown earlier, instantaneous electromagnetic torque can be expressed in the form of an infinite series containing function of machine parameters, inverter input voltage and operating frequency. This makes it possible to grasp the overall characteristics of instantaneous torque. If the exact solution for motor current is known, then accurate solution for the above-mentioned torque can be obtained.

4.3.2 Steady Harmonic Torque

Constant or steady harmonic torques are developed by the reaction of harmonic air gap fluxes with harmonic rotor currents of same order. However, these torques due to harmonics are very small fraction of the rated torque even at any speed and have negligible effect on motor operation. This is verified by calculating torque contribution of the motor from the expression of steady harmonic torque. Now, from equation (4.14),

$$t_{sh} = M_c \sum_{n=1,5,7}^{\infty} (i_{1nq} \cdot i'_{2nd} - i_{1nd} \cdot i'_{2nq}) \qquad (4.17)$$

Substituting the expression for instantaneous stator and rotor nth harmonic currents from the equations (3.4a), (3.4b), (3.7a) and (3.7b) in the above equation

$$t_{sh} = 2\, M_c \cdot I_1^2 \sum_{n=1,5,7}^{\infty} K_{nq} \cdot K_{nd} \cdot \text{Sin}\, \alpha_{cn} \qquad (4.18)$$

Equation (4.18) shows that steady harmonic torque is independ nt of time and is constant for any particular speed of the motor.

Figure 4.1 shows typical steady harmonic torque versus fundamental slip characteristics for positive and negative sequence harmonics. It can be noted from the above characteristics that contribution to average torque due to fifth order harmonic current is 0.115% of the contribution due to fundamental currents at rated speed of 1176 r.p.m. Thus the effect of fifth harmonic current on average torque is negligible and for higher order harmonics this torque is observed to be even less. Another notable feature is that counter torques due to negative sequence harmonics (fifth, eleventh etc.) are opposed by the forward torque due to positive sequence harmonics (seventh, thirteenth, etc.). The combined effect of those steady harmonic torques due to harmonics, produce a very small negative torque opposing the average torque due to fundamental.

As slip due to positive and negative sequence harmonics decreases and increases respectively with the decrease in motor fundamental slip (Table 3.3), therefore positive sequence and negative sequence steady harmonic torque also decreases and increases respectively with the increase in motor speed.

Therefore, due to negligible effect of supply harmonics as stated in average torque, the effective average torque versus slip characteristics shown in Figure 4.2 is approximately equal to

the normal speed versus torque characteristics of the induction motor. Characteristics show that in the range of motor action between standstill and critical speed, the torque increases more than proportionally to the speed. It implies that within this range there is condition of instability in the sense that if the torque developed is greater than the resisting torque, the speed will continue to rise until the point of maximum torque has been reached. Beyond the point of maximum torque and up to synchronism the conditions become stable, for here any increase of load torque causes the motor to slow down and therefore automatically to develope greater torque to meet the load requirement.

In negative slip region, motor operates as a regenerating braking condition. It should be noted that the absolute value of critical slip, where maximum torque occurs, is identical for both the region. However, during regeneration due to high value of stator resistance maximum torque will have its highest absolute value. As stator resistance decreases, $|T_{gen(max)}|$ approaches to $|T_{mot(max)}|$ and the characteristics proximates to purely antisymmetric about the torque axis, which is clearly shown in the Figure 4.2, taking one intermediate value of stator resistance between normal and ideally zero.

It can be observed from Figure 4.3 that maximum torque does not depend upon rotor resistance (r_2). The slip at maximum torque increases directly with the rise in rotor

resistance i.e. fall in rotor time constant and maximum torque point shifts to the position of greater slip with the insertion of rotor resistance. A family of rheostatic speed torque characteristics with several values of rotor resistances are shown in Figure 4.3(a) for normal inverter output voltage and frequency. Figure 4.3(b) shows the same characteristics for stator resistance ideally to be zero.

In varying the inverter frequency, it is necessary to make the characteristics retain high hardness throughout the entire range of speed control and allow the motor to show adequate overload torque capacity. This may be achieved by causing each motor to operate with its magnetic flux maintained constant. A set of average torque versus slip characteristics controlled by varying the fundamental of inverter output voltage with proportional change in operating frequency can be seen from Figure 4.4. Figure 4.4(a) shows the above set of characteristics for normal machine parameters. But it is found in Figure 4.4(b) that maximum torque within the frequency range (30 Hz to 60 Hz) remains unchanged as r_1 is made ideally to be zero provided air gap flux maintained constant in the machine.

At low frequencies, due to relative rise in the influence of the voltage drop in stator, the magnetic field undergoes a significant decrease in strength. As a consequence, maximum torque drops to a lower value as shown in Figure 4.4(c). Figure 4.4(d) shows the above low frequency average torque slip

characteristics for ideally zero stator resistance.

So it is advisable to maintain constant flux operation at the lower most value of inverter frequency. The leakage reactance drops to a value commensurable with and even less than that of stator resistance. Besides maintaining the motor overload torque capacity sufficiently high at low frequencies, it is desired to decrease the voltage by a lesser degree than the frequency.

4.3.3 Pulsating Harmonic Torque

Pulsating harmonic torque with a mean value of zero are produced by the reaction of harmonic rotating fluxes with harmonic rotor currents of different order. From equation (4.14)

$$t_{ph} = M_c \sum_{h=6,12,18}^{\infty} \Big[\sum_{\substack{m=1,5,7 \\ n=h-m}}^{(h-1)} (i_{1mq} \cdot i_{2nd} - i_{1nd} \cdot i_{2mq})$$

$$+ \sum_{\substack{m=1,5,7 \\ n=h+m}}^{\infty} \{ (i_{1mq}, i_{2nd} \cdot i_{2mq}) + (i_{1nq} \cdot i_{2md} - i_{1md} \cdot i_{2mq})) \} \Big] \qquad (4.19)$$

The principal pulsatory torque arise from the interaction between the fundamental rotating flux and harmonic rotor current and vice versa. The rotor currents induced by the time harmonics field, react with fundamental rotating flux. This produces a pulsating torque at six times the fundamental

frequency. This is the torque which is responsible for severe fluctuation in the electromagnetic torque developed by the motor specially at low speed region.

Figure 4.5 represents the instantaneous pulsating harmonic torque of the induction motor fed by six step voltage source inverter for varying speed under normal operating frequency of 60 Hz. It is observed that the waveforms repeat after every $\frac{\pi^c}{3}$ interval which is analogous to the results obtained in [12]. Again it is also noticable that though at synchronous speed fundamental torque is zero, but due to the predominancy of the harmonic components only pulsating harmonic torque exists.

In Figures 4.6 to 4.8 effect of low frequencies on instantaneous pulsating harmonic torque is shown keeping the ratio of magnitude of fundamental component of inverter output voltage to inverter operating frequency constant. Here it is seen that as speed increases to synchronous speed the pulsation of the above-mentioned torque becomes more and more. The ripple in the instantaneous pulsating harmonic torque at low frequencies is similar to that of the ripple torque of the induction motor driven by current source inverter [20].

Figure 4.9 shows the approximate average sixth harmonic torque as a combined effect of the principal sixth harmonic torque [$t_{p(a,5)}$ and $t_{p6(1,7)}$] for different values of fundamental slip.

It has been found that as stator resistance decreases, peak value of the torque in the motoring and generating region respectively increases and decreases, which is well understood in the above said figure.

Figure 4.10(a) has been drawn to compare the above sixth harmonic torque with the second predominant twelfth harmonic torque. The effect of stator resistance on both the characteristics has also been shown in Figure 4.10(b).

In Figure 4.11(a), two predominant components of sixth harmonic torque have been shown separately to know the effect of individual current harmonic components. The same characteristics were shown in Figure 4.11(b) taking the stator resistance of the motor to be ideally zero. It has been found that absolute value of critical slip where maximum sixth harmonic torques occurs is identical for both the positive and negative fundamental slip regions in case of $r_1 = 0$.

Again to observe the effect of individual phase angles of principal sixth harmonic torques on the resultant sixth harmonic torque, the magnitudes as well as phase angles variation of the principal sixth harmonic torques and their resultant are plotted for different value of fundamental slip ranging from (-1) to (+1). It is noticed that, at close to synchronous speed, the individual phase angle of sixth harmonic torques are zero and π^c corresponding to positive and negative sequence harmonic components of motor current, whereas resultant

the above polar plot of sixth harmonic torque to be approximately symmetric about the abscissa shown in Figure 4.12(b).

Figure 4.13 shows the effect of variation of rotor resistance on sixth harmonic torque versus fundamental slip characteristics. With increasing rotor resistance, the maximum motoring torque point not only shifts to the higher side of the fundamental slip but also decreases almost linearly. However, in the negative fundamental slip region, maximum torque point shifts to the higher side of the fundamental slip with a gradually increasing value.

Figure 4.14(a) depicts the pulsating sixth harmonic torque variation with motor fundamental slip for different inverter operating frequency with ($\sqrt{2}\, V_{dc}/\pi . f_s$) ratio as constant. It is observed that for maintaining air gap flux constant throughout all the frequencies the maximum motoring torque decreases as inverter operating frequency decreases due to the same reason as stated in section 4.3.1 for average torque. Effect of ideally zero stator resistance has been shown in Figure 4.14(b). Here, maximum motoring torque decreases with a lesser degree as compared with the Figure 4.14(a).

Sixth harmonics torque versus fundamental slip characteristics for low frequencies are plotted in Figure 4.14(a). The followings are observed-

At low speed regions the above torques are not affected by inverter operating frequency.

However, at close to synchronous speed region, sixth harmonic torque decreases with the fall in supply frequency.

The above family of characteristics are again drawn in Figure 4.14(b) for ideally zero stator resistance. Here, one special observation is that for frequencies above 4 Hz, sixth harmonic torque at close to synchronous speed becomes lower than the torques at lower speed ranges.

4.4 EFFECT OF HARMONIC TORQUES IN INDUCTION MOTOR

However in general there is no alteration in the steady state torque of the motor since the pulsating torques have zero average value but their presence causes the angular velocity of the rotor to vary during revolution. At very low speeds, the motor rotation takes place in a series of jerks or steps and the presence of the pulsating torques may set a lower limit to the useful speed range of the motor. The point at which the speed pulsation becomes objectionable in a variable speed drive depends on the inertia of the rotating system and nature of the application.

The induction motor operated with nonsinusoidal supply has the usual motor losses and some additional losses due to the harmonics. The losses are as follows:

(i) Stator winding loss: Besides the usual stator I^2R loss, with an addition term to account for the loss due to

harmonic currents.

(ii) Rotor winding loss: Though it is negligible in V.S.I. fed induction motor,but to overcome the small negative torque produced by the harmonic currents it increases with slip speed.

(iii) Core loss: It increases slightly due to the effect of higher peak flux density caused by magnetizing current.

(iv) Friction and Windage Loss: Not influenced in any way.

(v) Rotor Harmonic Loss: It increases with harmonic currents if variation of rotor resistance due to frequency is considered.

(vi) Stray Load Loss: Increases with harmonics content in motor currents.

[14]

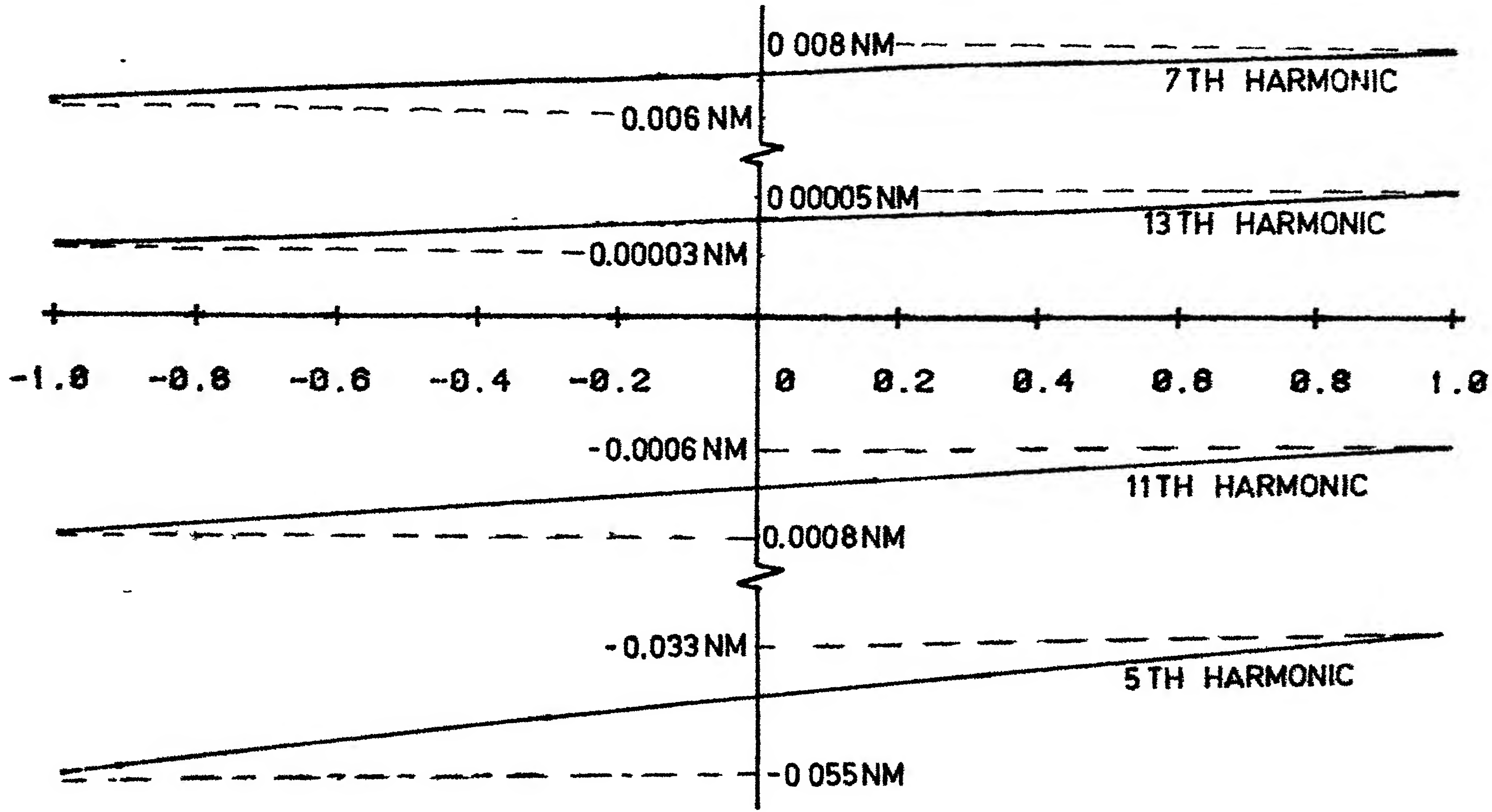


FIG 4.1 TYPICAL STEADY HAR. TORQUE VS. FUND. SLIP CHARACTERISTICS.

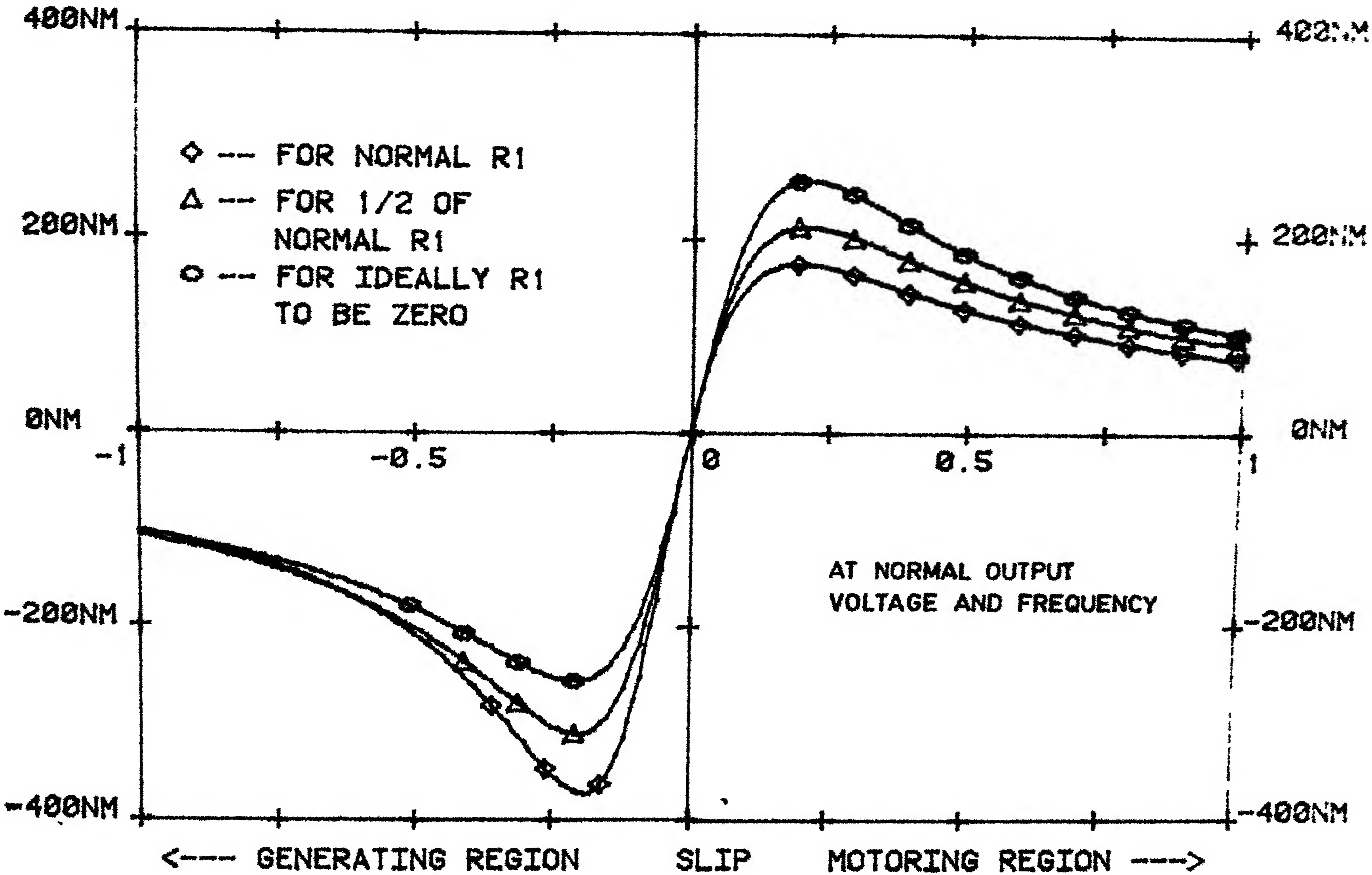


FIG 4.2 AVERAGE TORQUE SLIP CHARACTERISTICS SHOWING THE EFFECT OF R1.

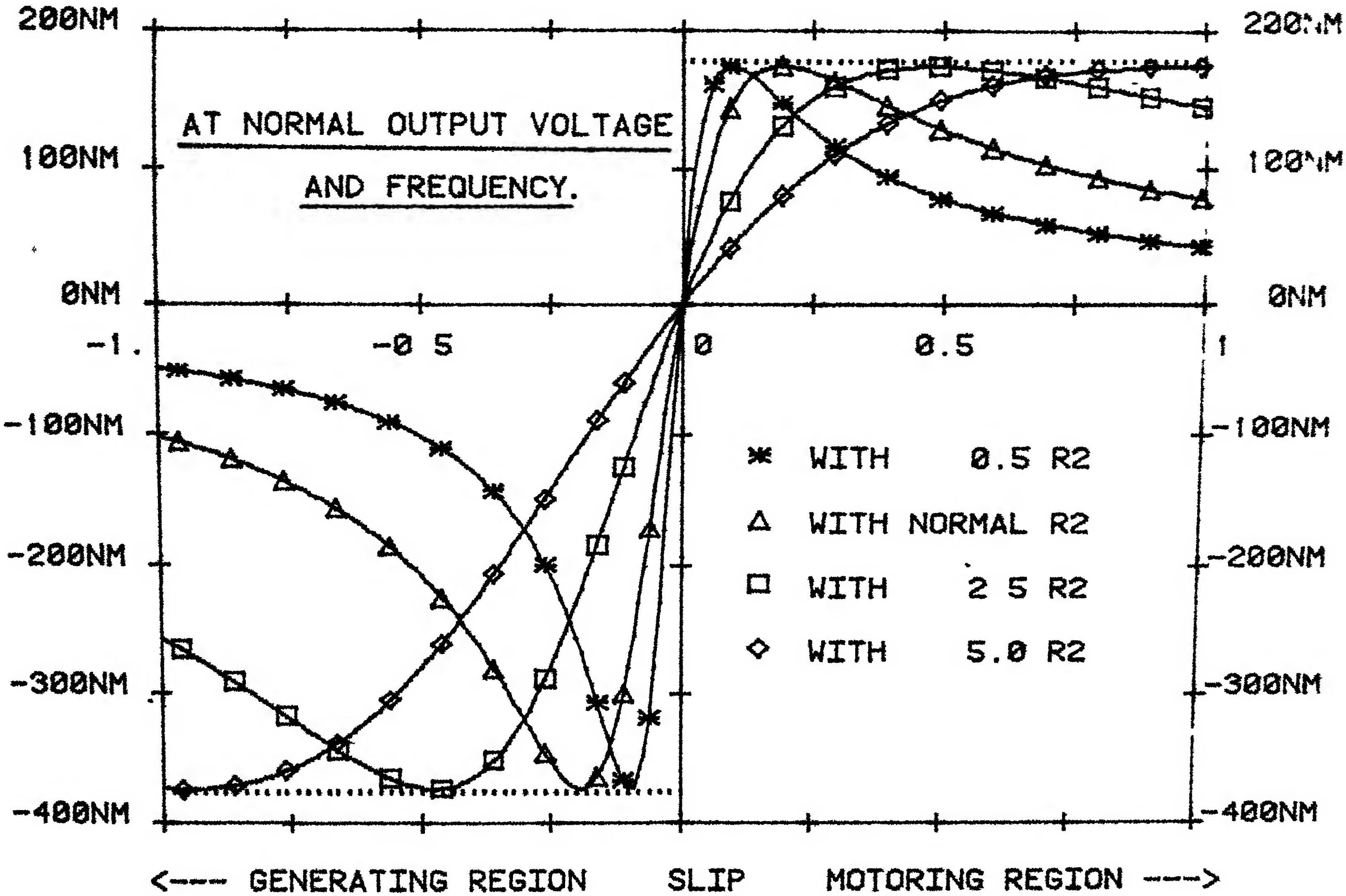


FIG 4.3(A) RHEOSTATIC AVERAGE TORQUE SLIP CHARACTERISTICS

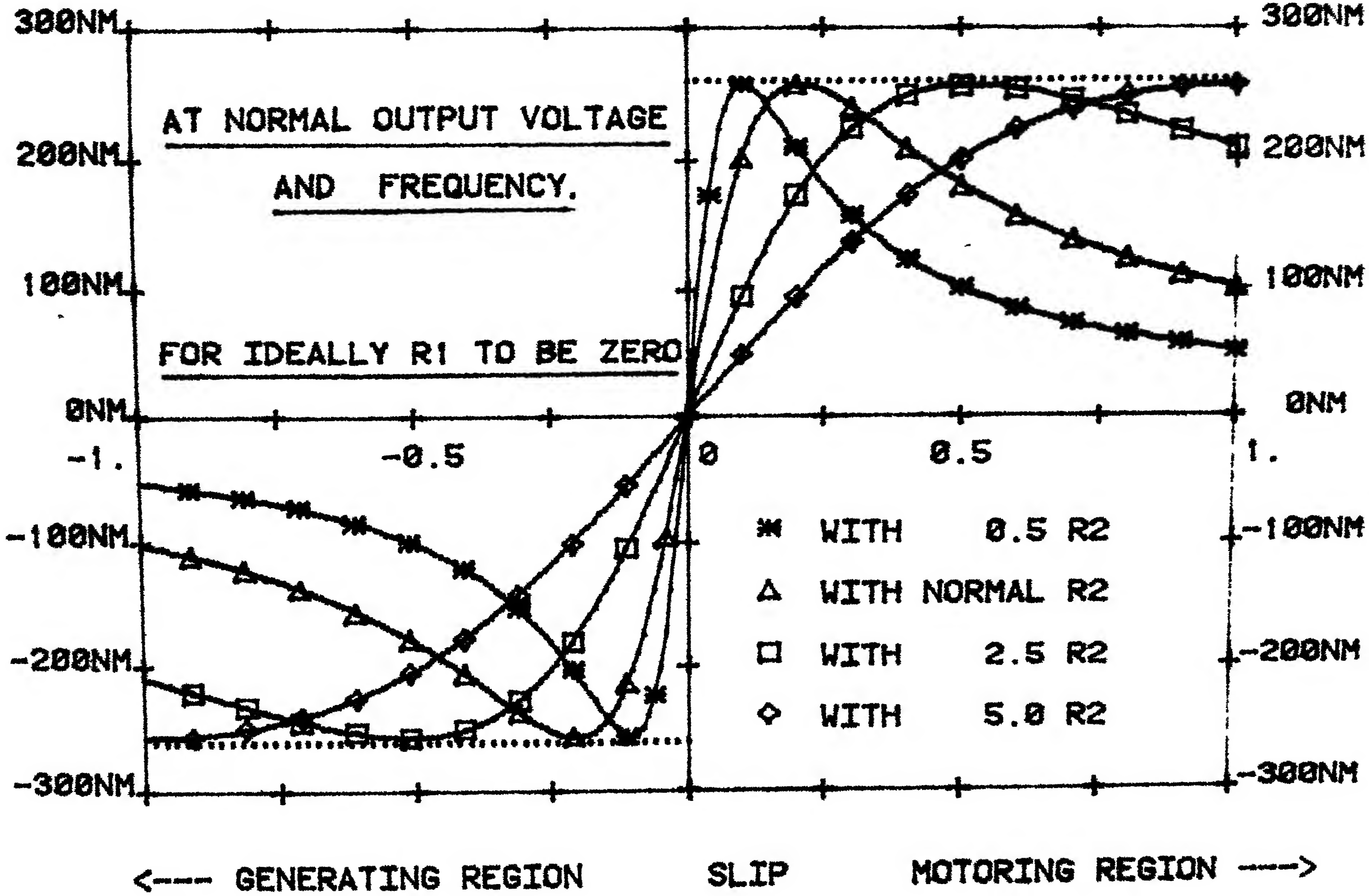


FIG 4.3(B) RHEOSTATIC AVERAGE TORQUE SLIP CHARACTERISTICS.

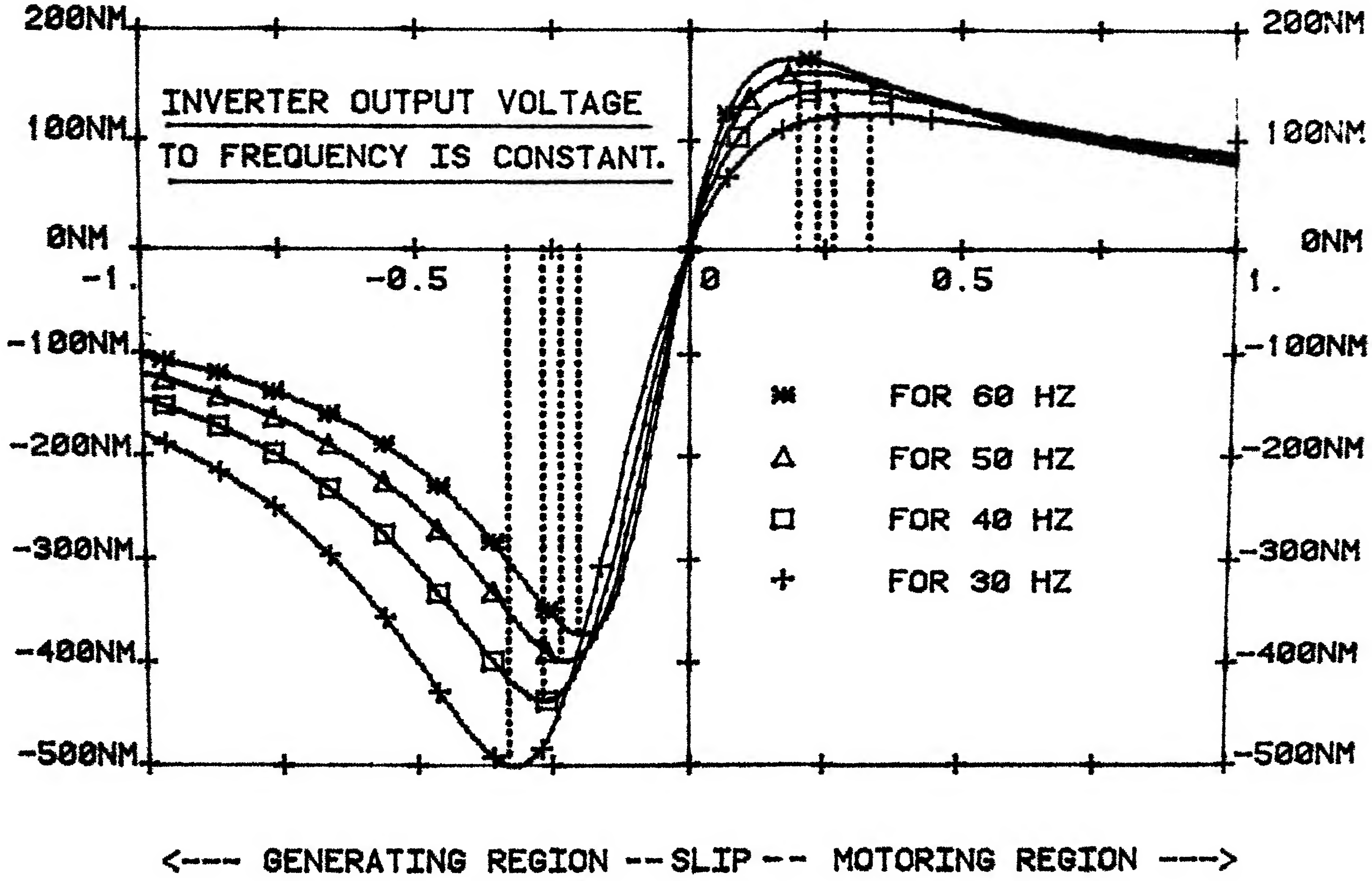


FIG 4.4(A) AVERAGE TORQUE SLIP CHARACTERISTICS FOR VARING INVERTER FREQ.

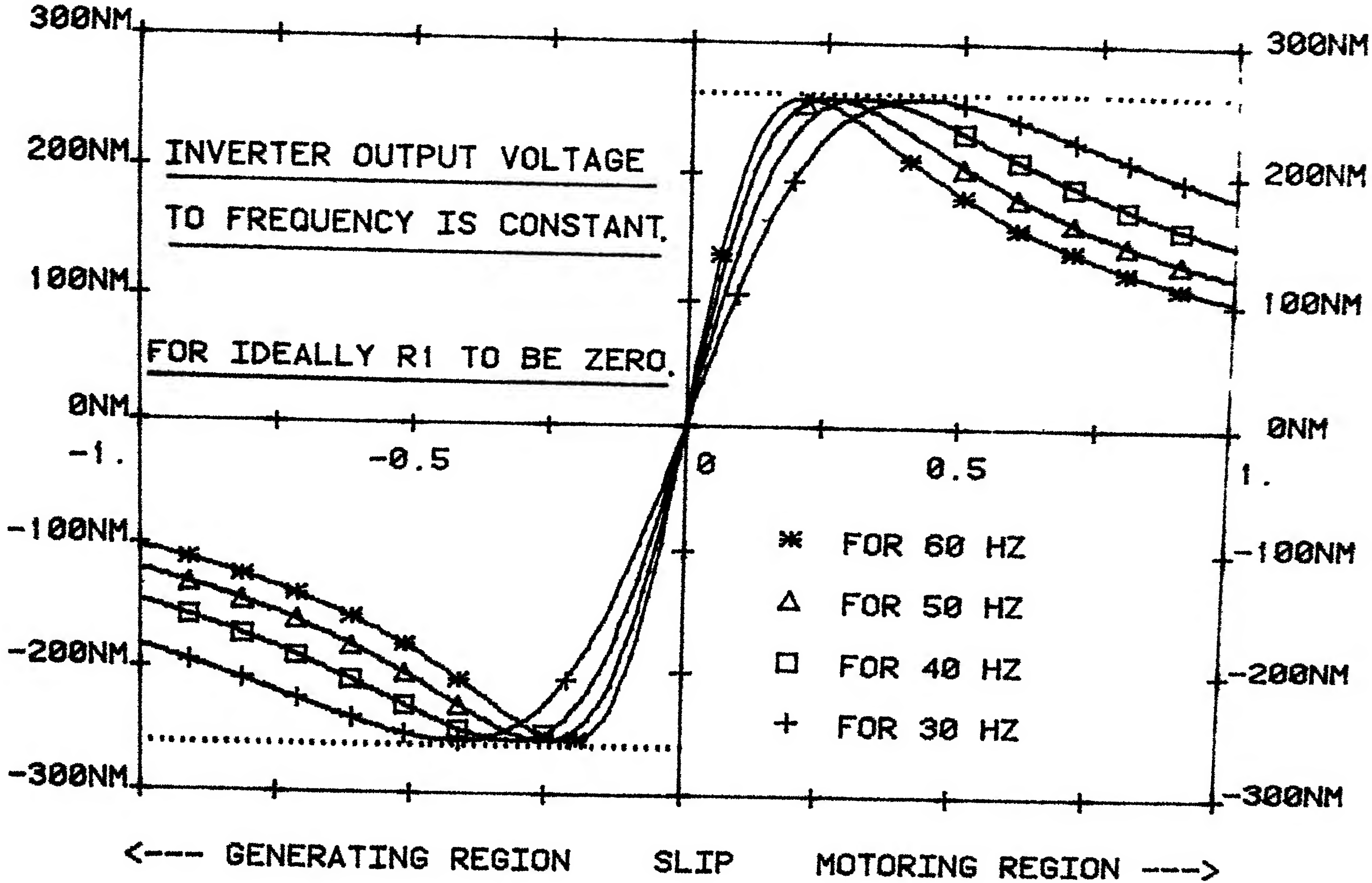


FIG 4.4(B) AVERAGE TORQUE SLIP CHARACTERISTICS FOR VARING INVERTER FREQ.

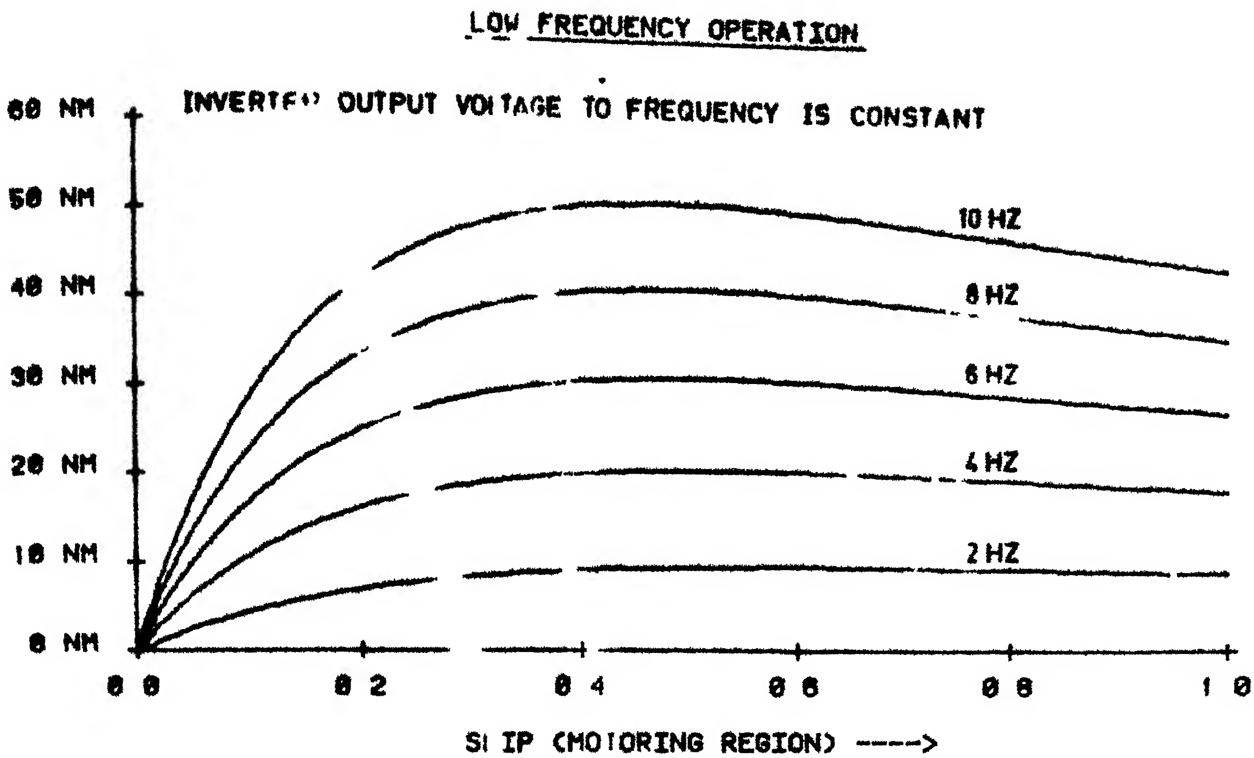


FIG 4.4(C) AV TORQUE SLIP CHARACTERISTICS FOR VARING INVERTER FREQUENCY

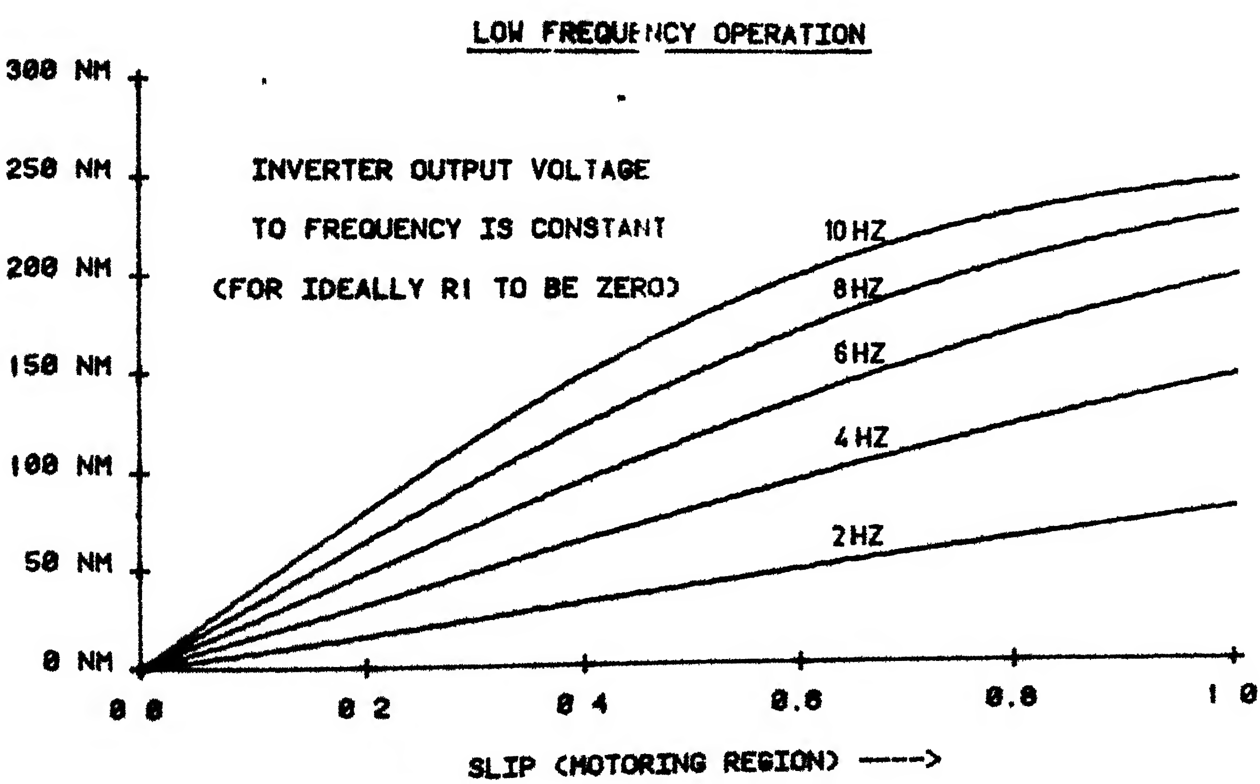


FIG 4.4(D) AV TORQUE SLIP CHARACTERISTICS FOR VARING INVERTER FREQUENCY

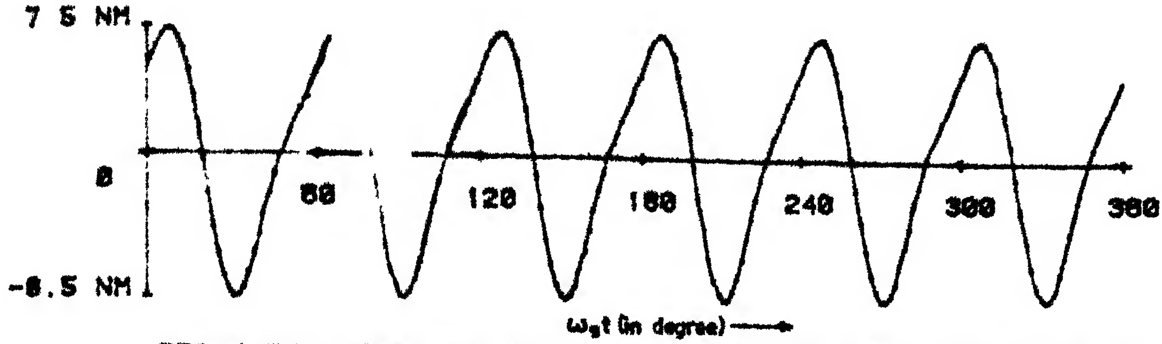


FIG 4.5(A) INST HAR TORQUE AT SPEED 600 R P.M (FREQ=60 HZ)

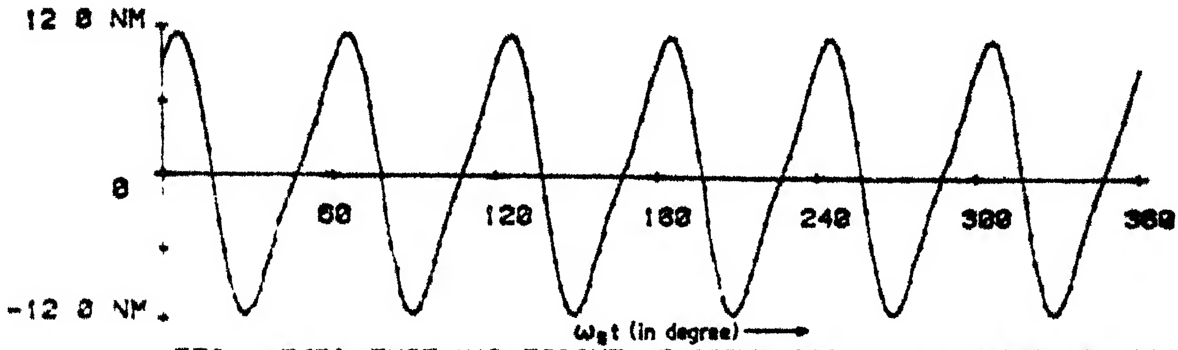


FIG 4 5(B) INST. HAR TORQUE AT SPEED 960 R P M (FREQ=60 HZ)

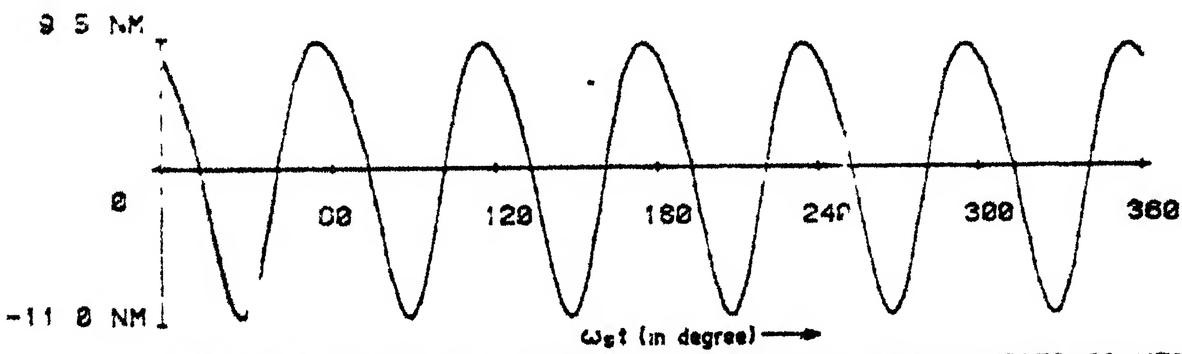


FIG 4 5(C) INST HAR TORQUE AT SPEED 11[illegible] R P M (FREQ=60 HZ)

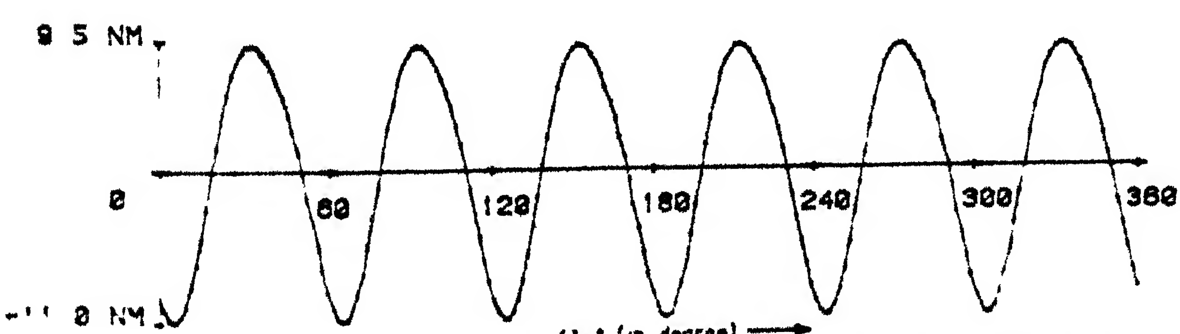


FIG 4.5(D) INST HAR TORQUE AT SPEED 1200 R P M (FREQ=60 HZ)

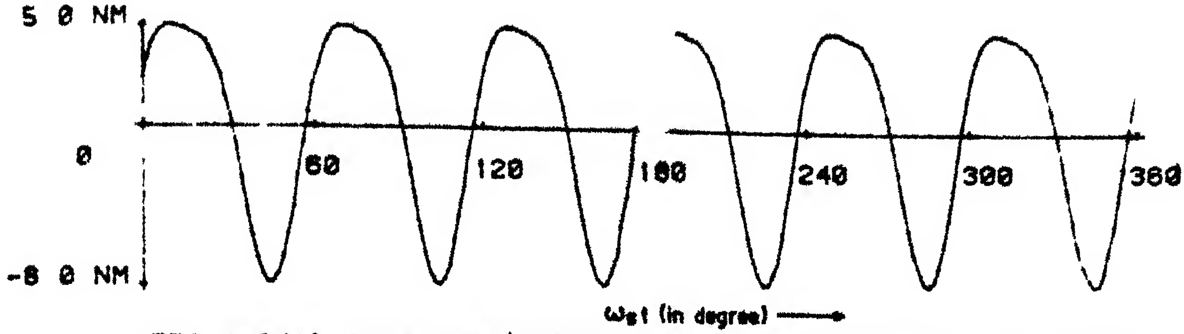


FIG 4.6(A) INST HAR TORQUE AT SPEED 300 R P M (FREQ=30 HZ)

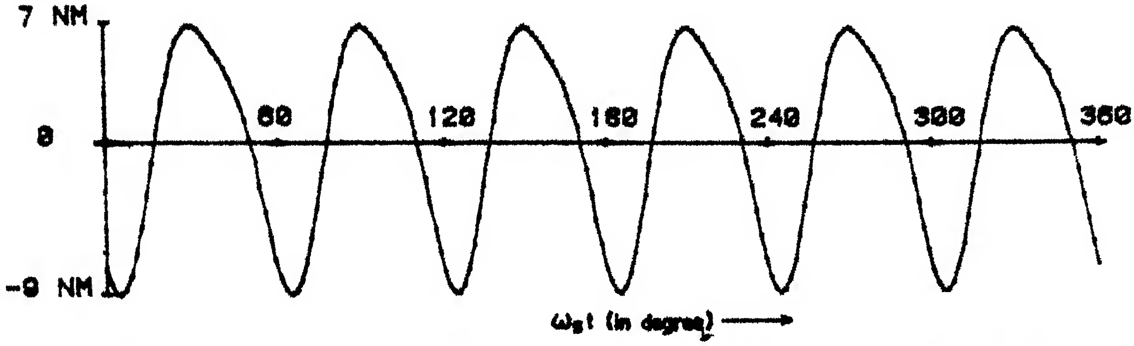


FIG 4.6(B) INST HAR TORQUE AT SPEED 400 R P M (FREQ=30 HZ)

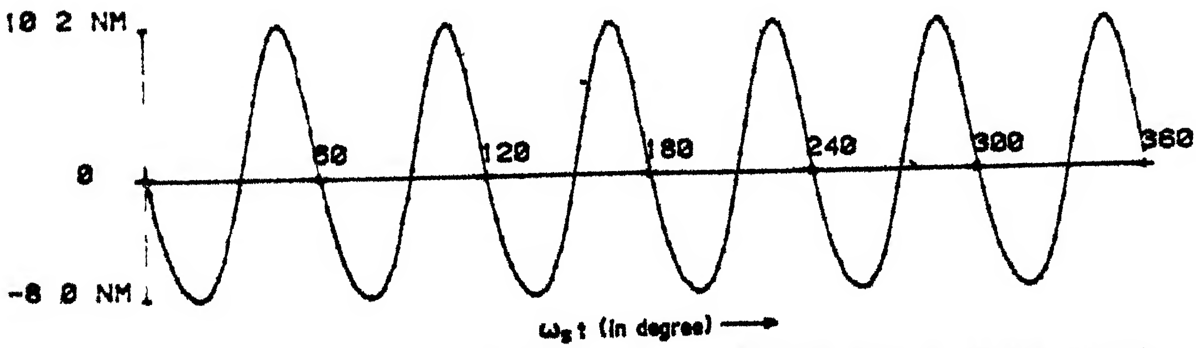


FIG 4.6(C) INST HAR TORQUE AT SPEED 500 R P M (FREQ=30 HZ)

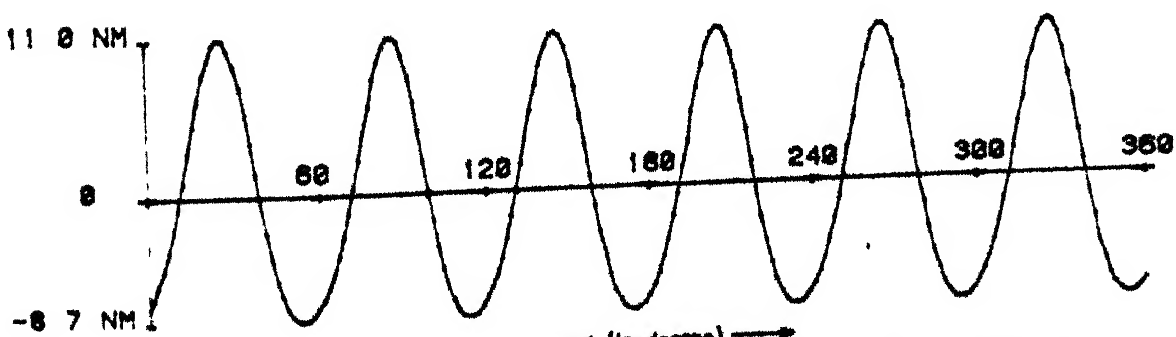


FIG 4 6(D) INST HAR TORQUE AT SPEED 600 R P M (FREQ=30 HZ)

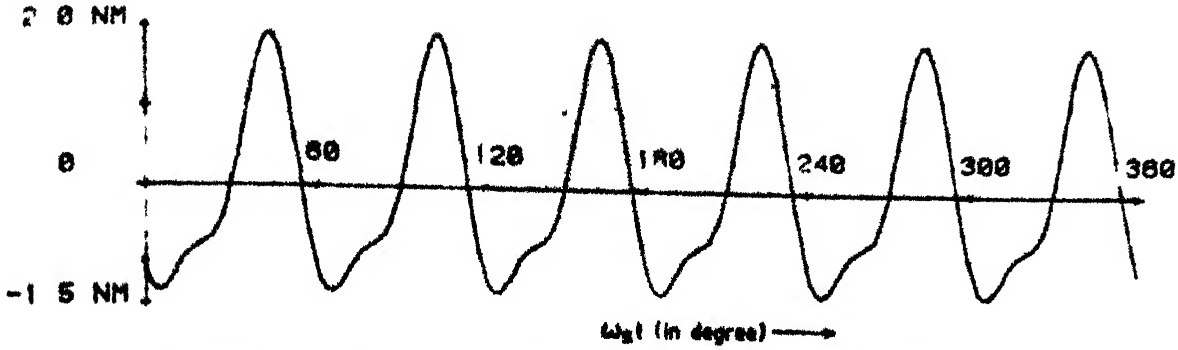


FIG 4.7(A) INST HAR TORQUE AT SPEED 30 R P M (FREQ=3 HZ)

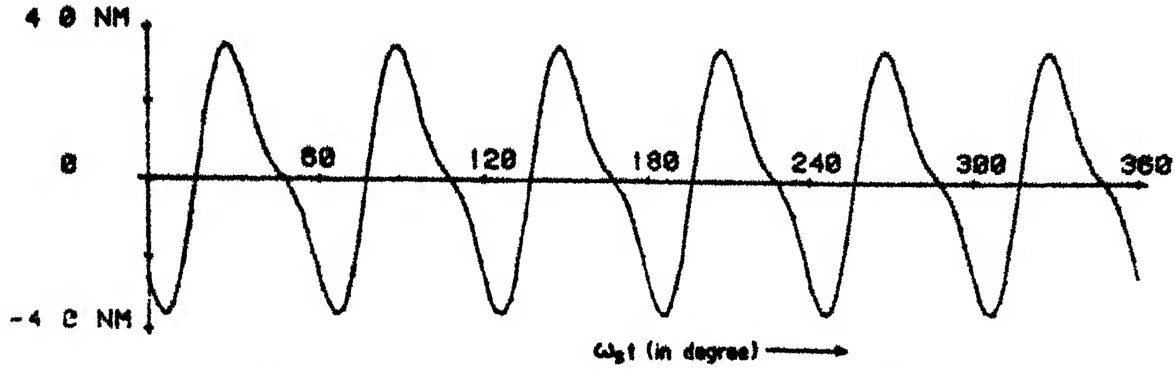


FIG 4.7(B) INST HAR TORQUE AT SPEED 40 R P M (FREQ=3 HZ)

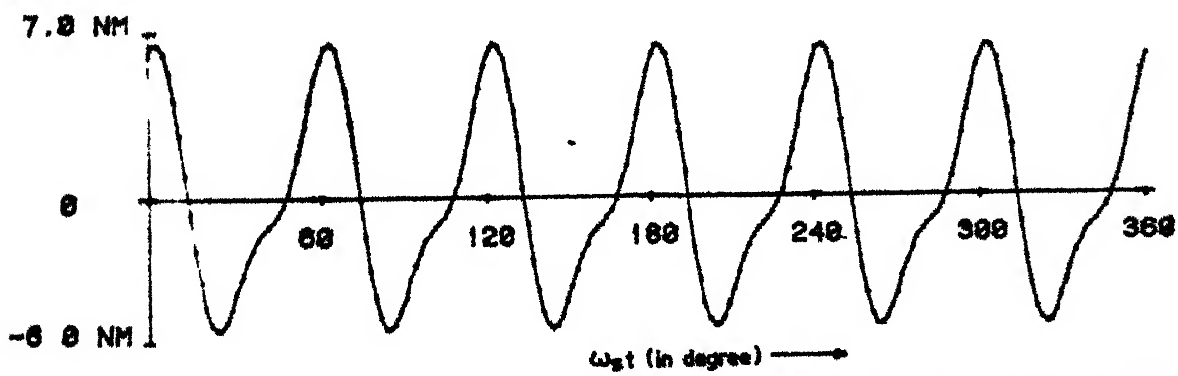


FIG 4.7(C) INST HAR. TORQUE AT SPEED 50 0 R P M (FREQ=3 HZ)

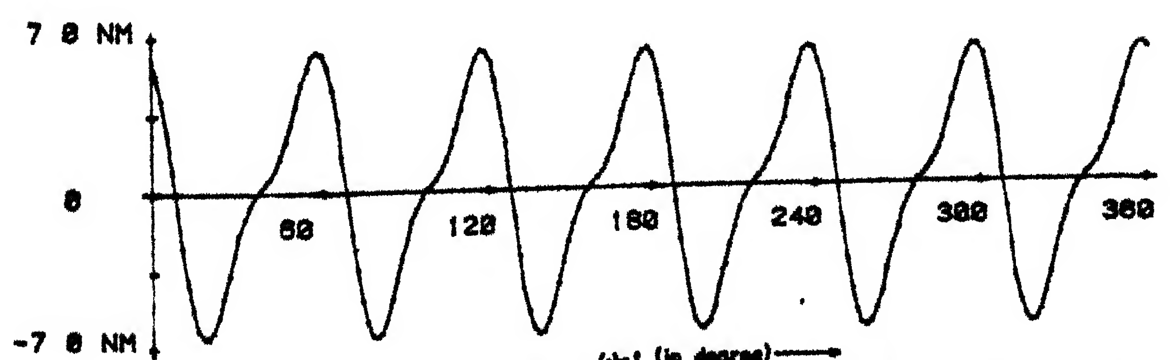


FIG 4.7(D) INST HAR TORQUE AT SPEED 60 R P.M (FREQ=3 HZ)

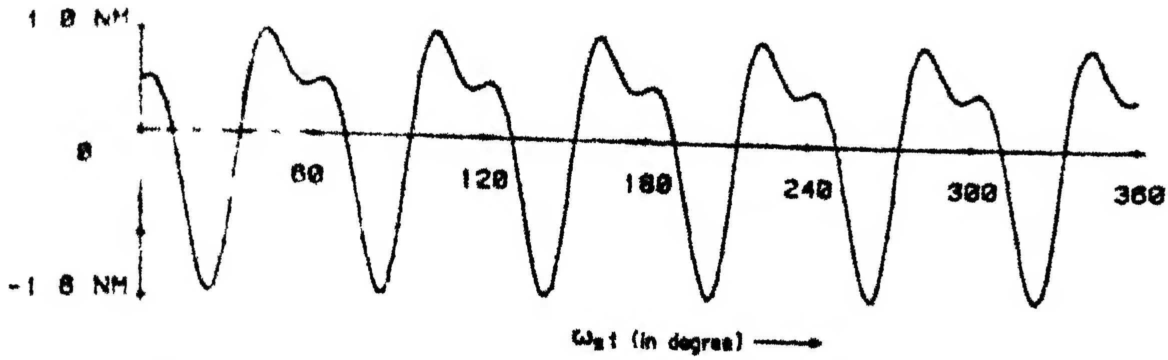


FIG 4 8(A) INST HAR TORQUE AT SPEED 10 R P M (FREQ=1 HZ)

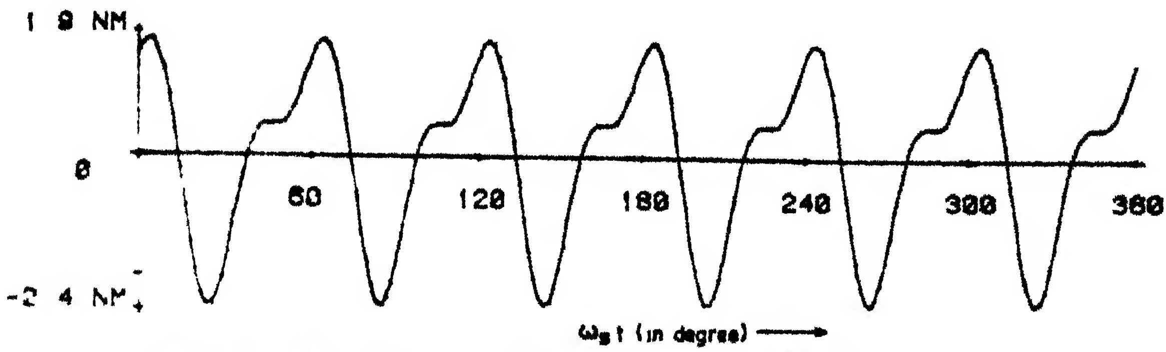


FIG 4 8(B) INST HAR TORQUE AT SPEED 16 R P M (FREQ=1 HZ)

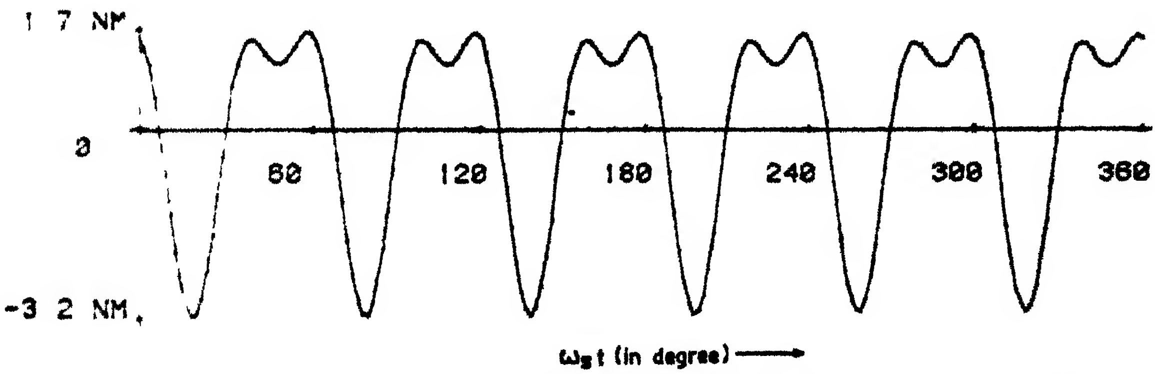


FIG 4.8(C) INST HAR TORQUE AT SPEED 19 6 R P M (FREQ=1 HZ)

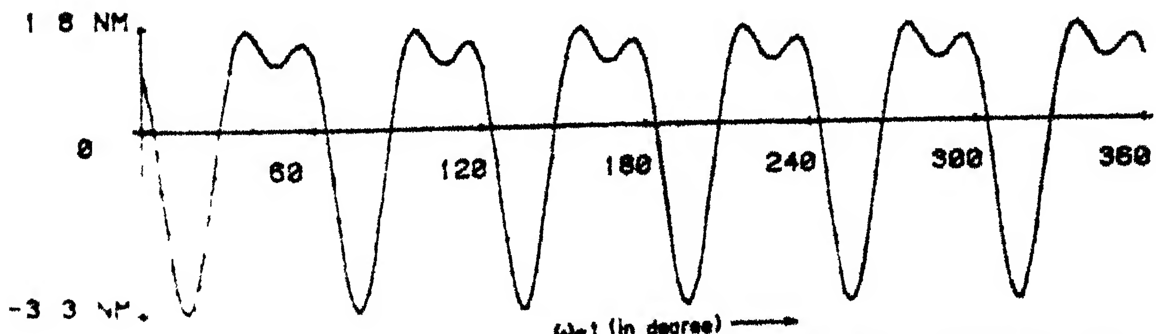


FIG 4 8(D) INST HAR TORQUE AT SPEED 20 R P M (FREQ=1 HZ)

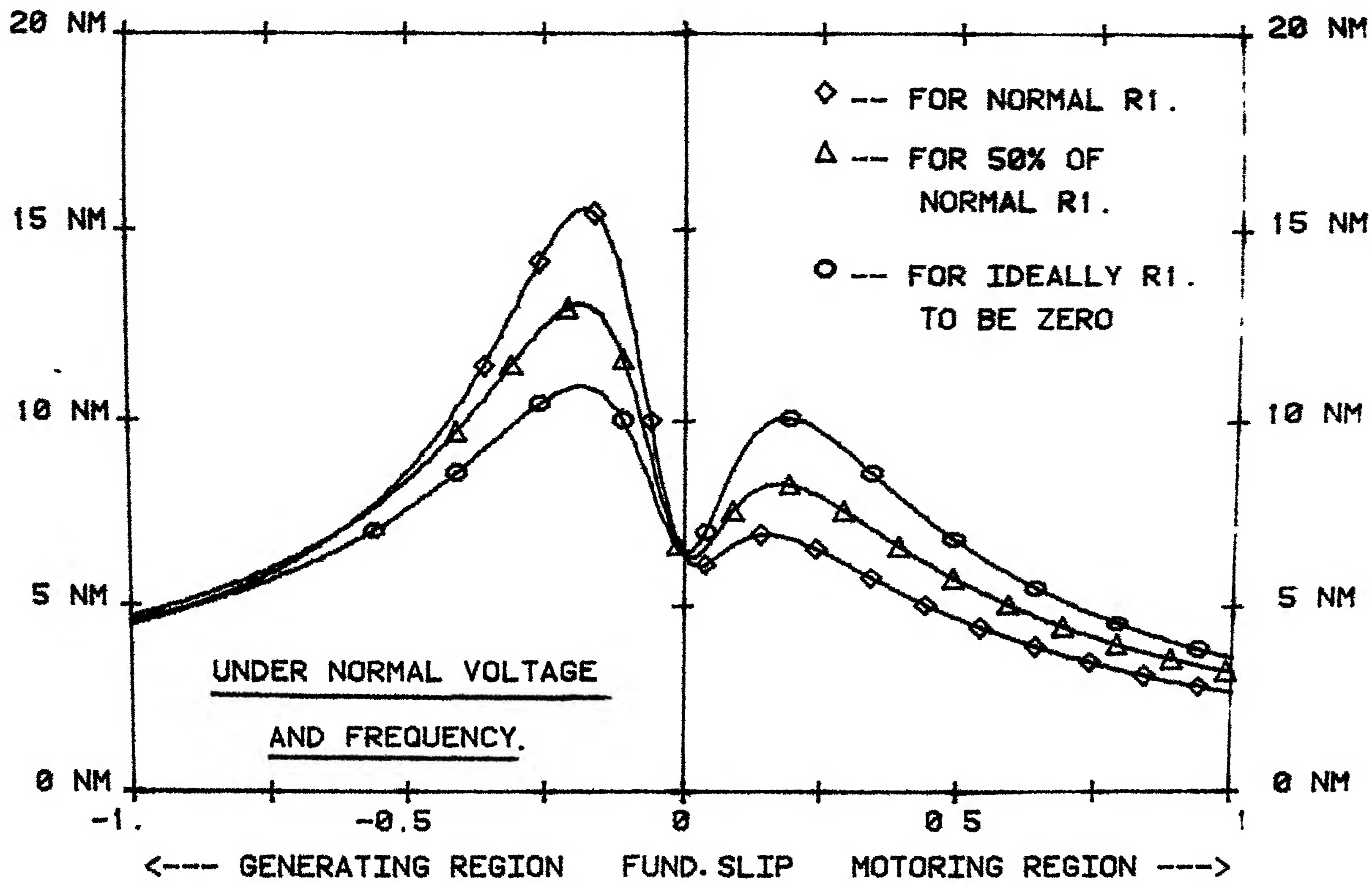


FIG 4.9 SIXTH HARMONIC TORQUE SLIP CHARACTERISTICS SHOWING THE EFFECT OF R1

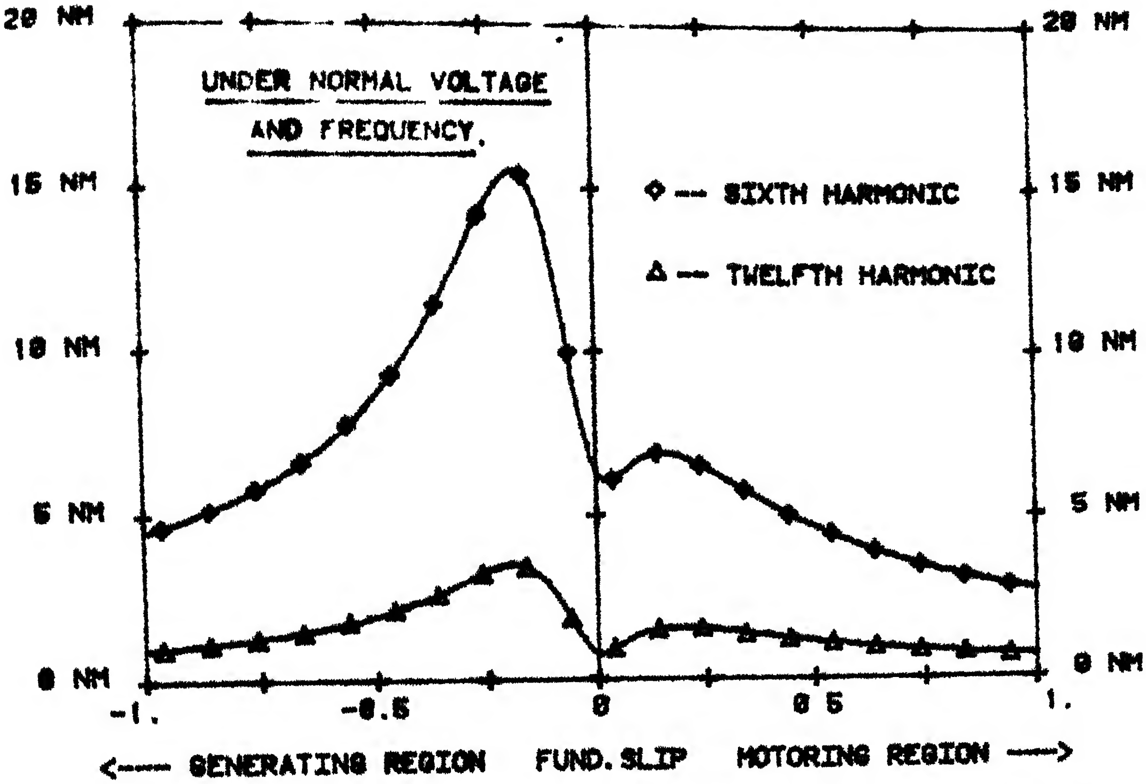


FIG 4.10(A) PULSATING HARMONIC TORQUE SLIP CHARACTERISTICS

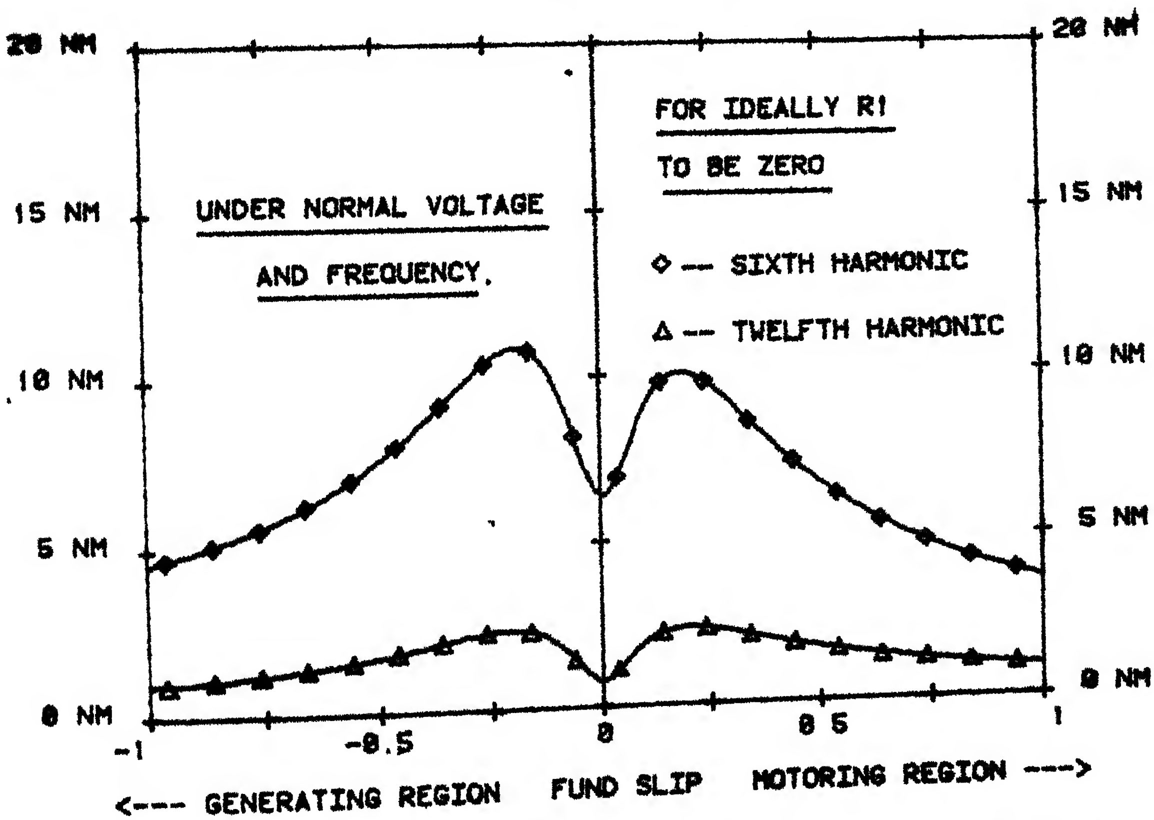


FIG 4.10(B) PULSATING HARMONIC TORQUE SLIP CHARACTERISTICS

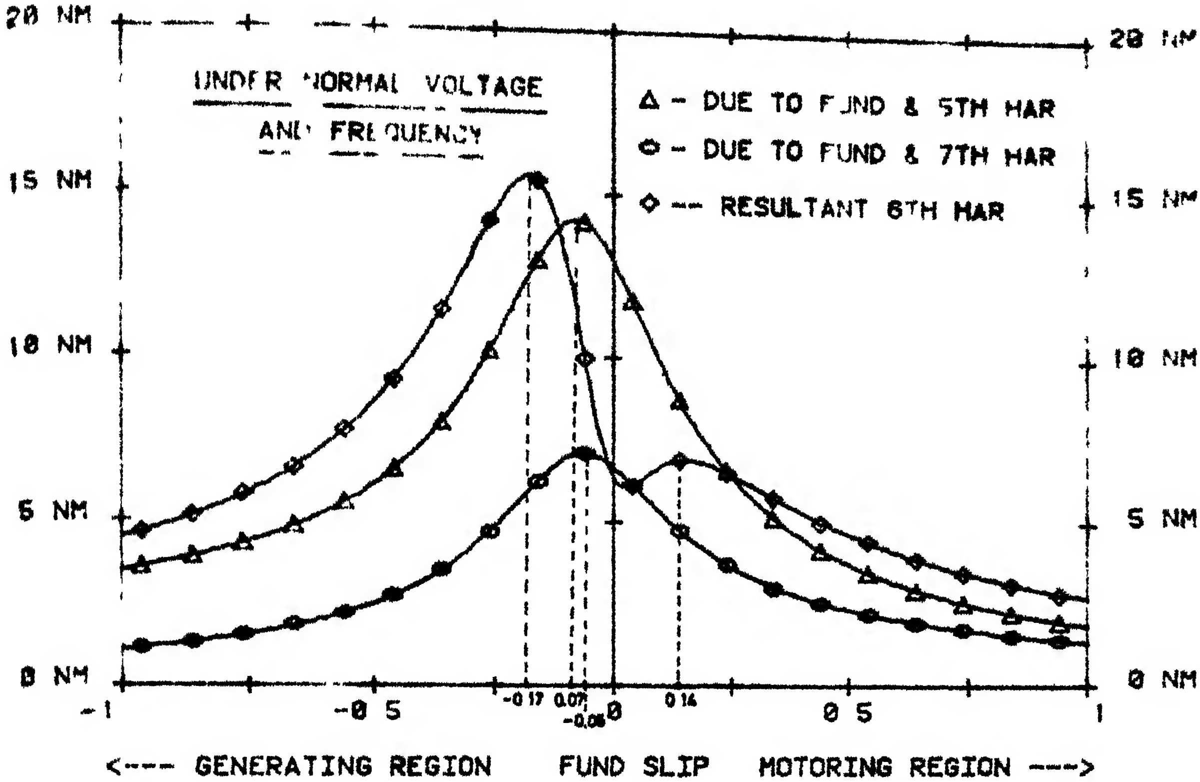


FIG 4.11(A) SIXTH HARMONIC TORQUE SLIP CHARACTERISTICS

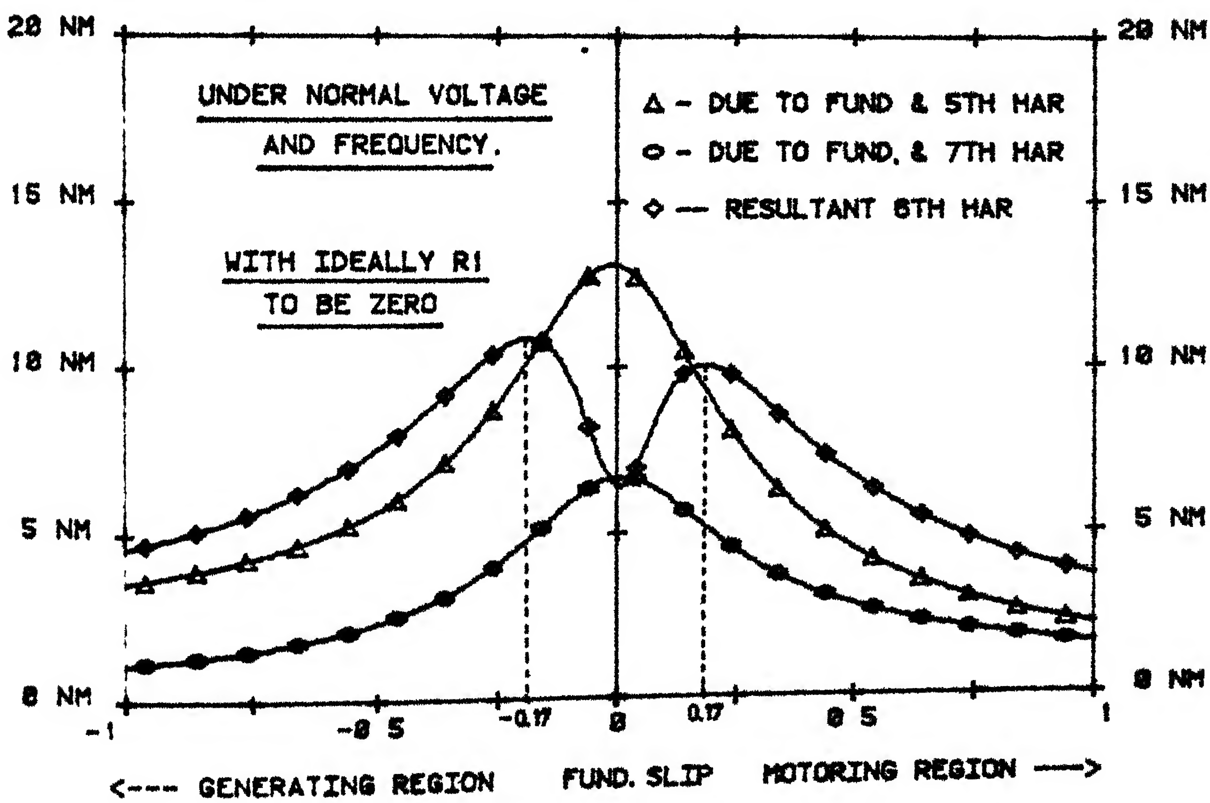


FIG 4.11(B) SIXTH HARMONIC TORQUE SLIP CHARACTERISTICS

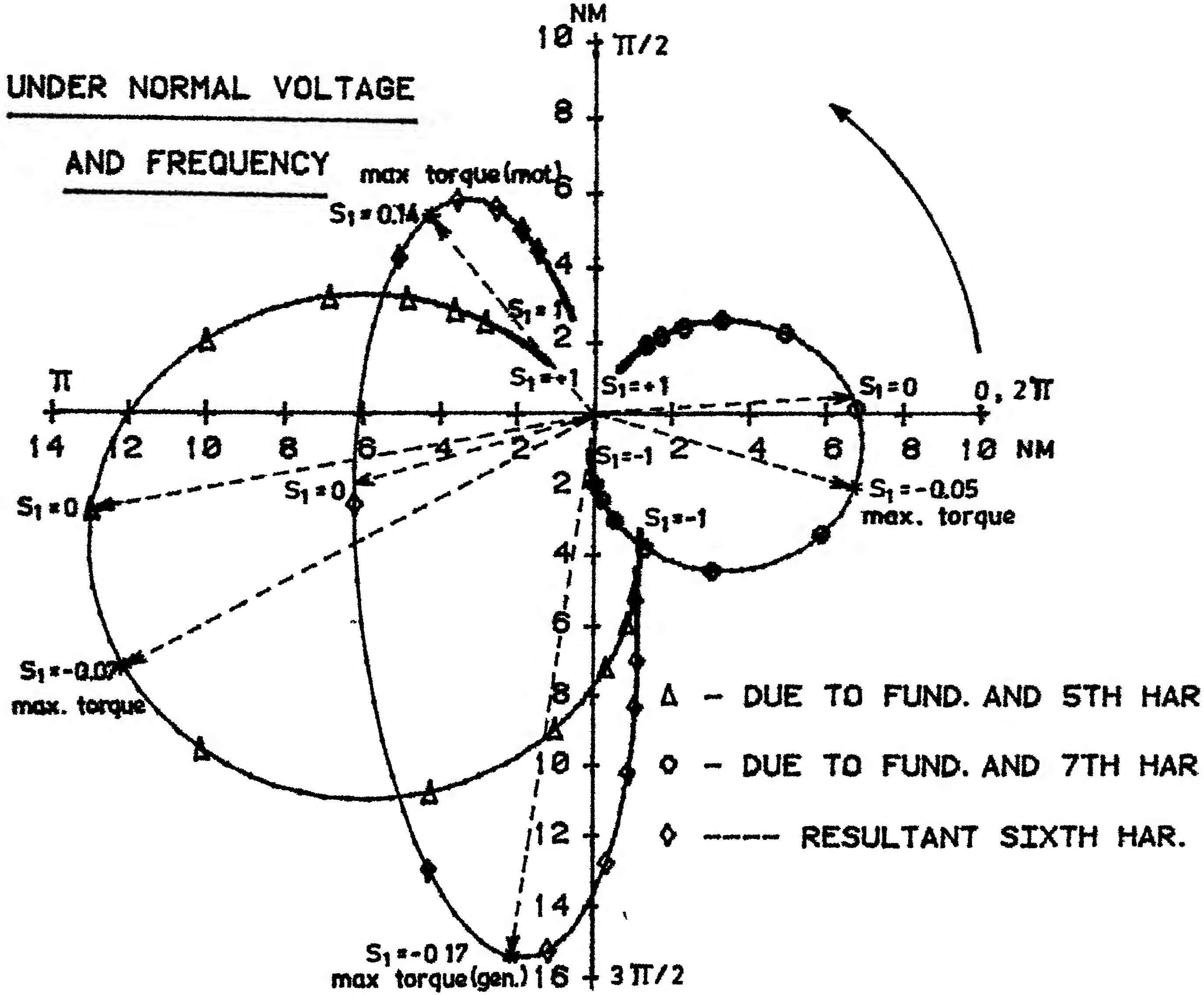


FIG 4.12(A) POLAR PLOT OF SIXTH HARMONIC TORQUE

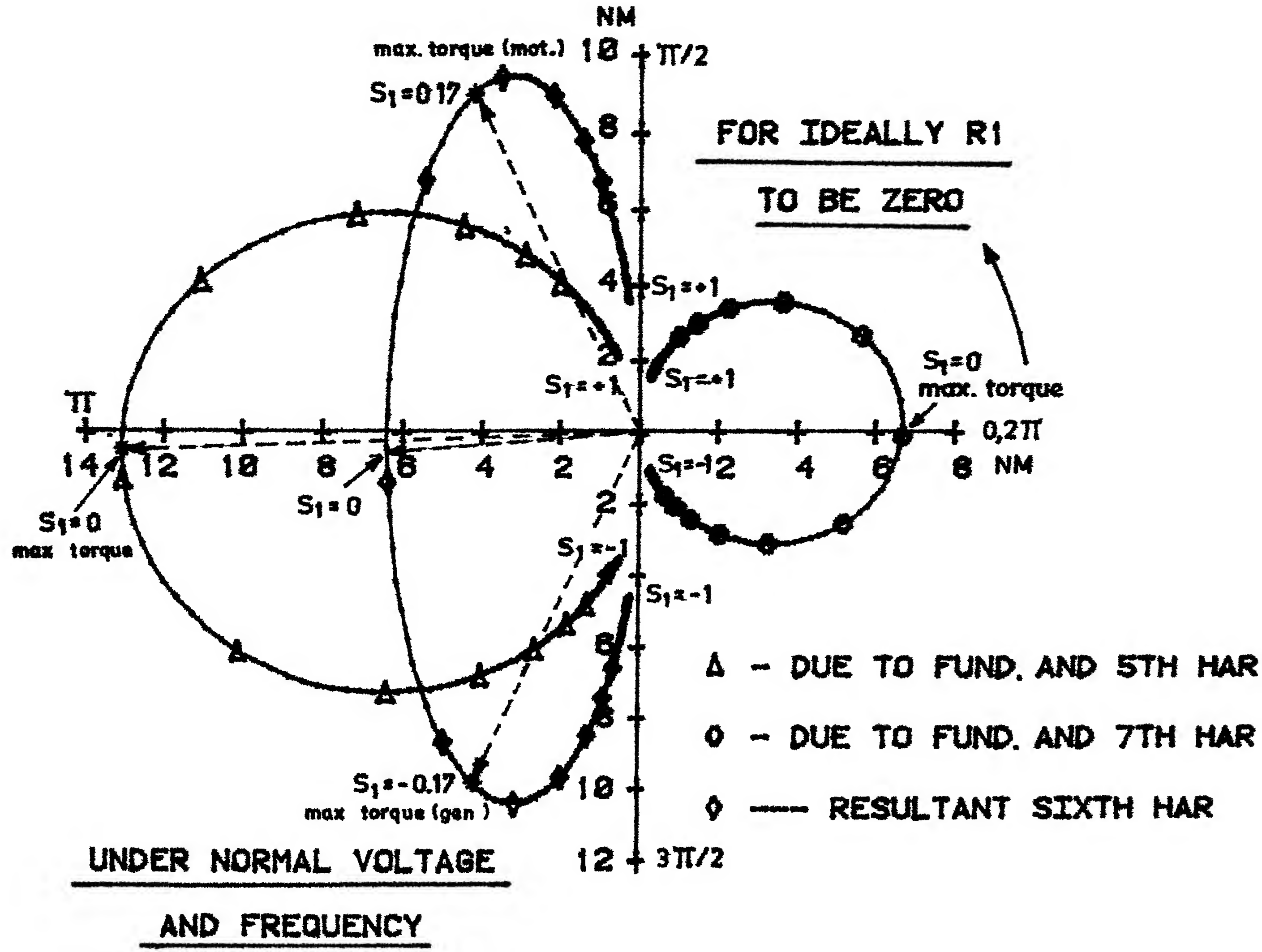


FIG 4.12(B) POLAR PLOT OF SIXTH HARMONIC TORQUE

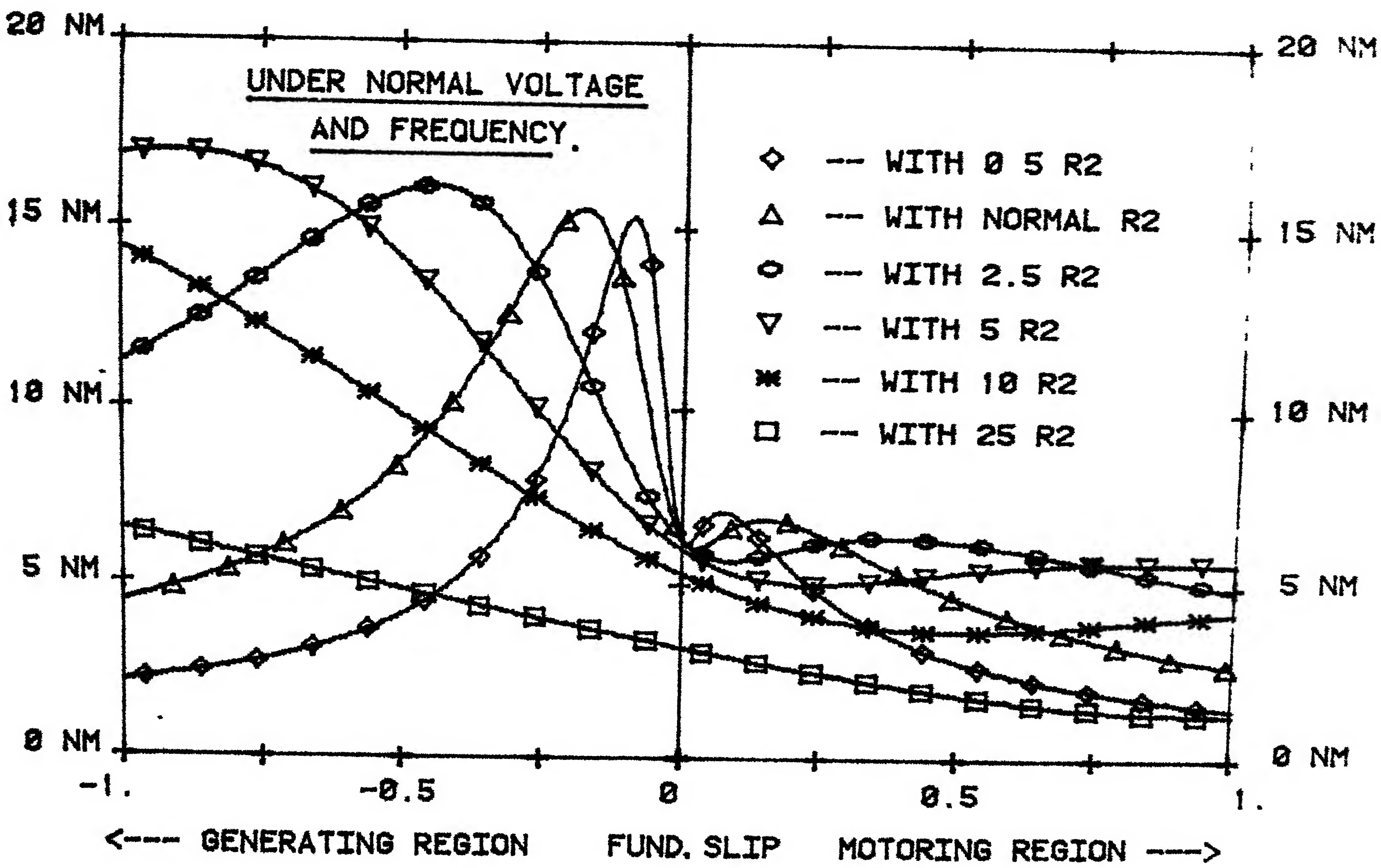


FIG 4.13(A) SIXTH HARMONIC TORQUE SLIP CHARACTERISTICS.

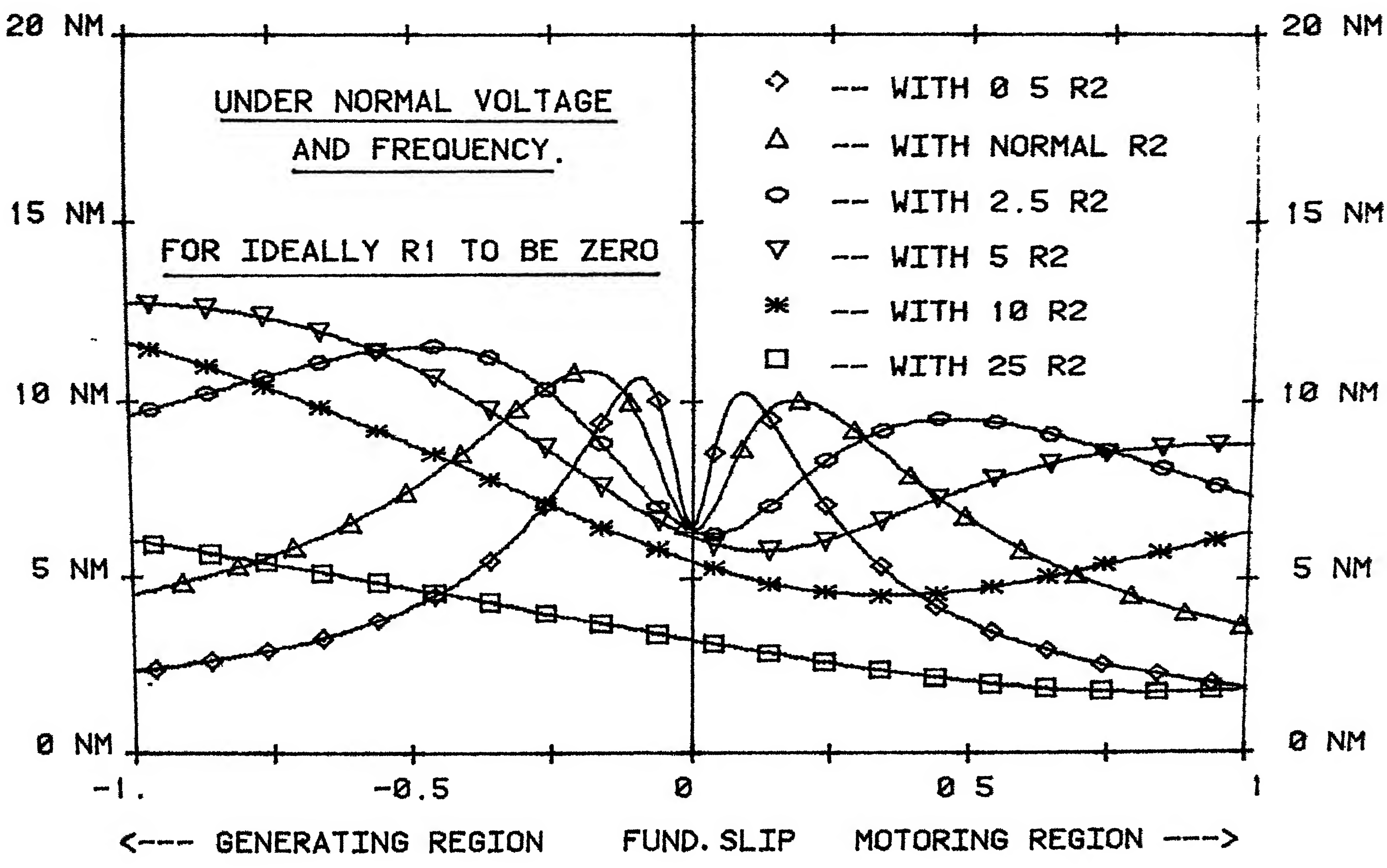


FIG 4.13(B) SIXTH HARMONIC TORQUE SLIP CHARACTERISTICS.

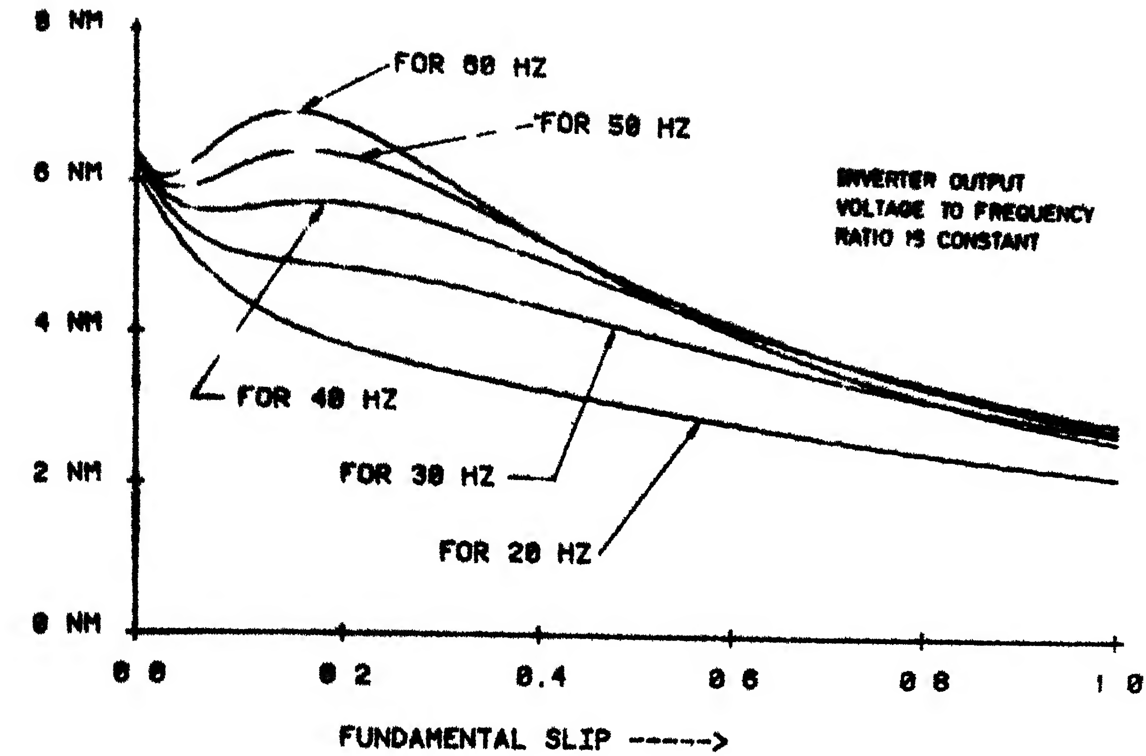


FIG 4 14(A)   SIXTH HARMONIC TORQUE SLIP CHARACTERISTICS

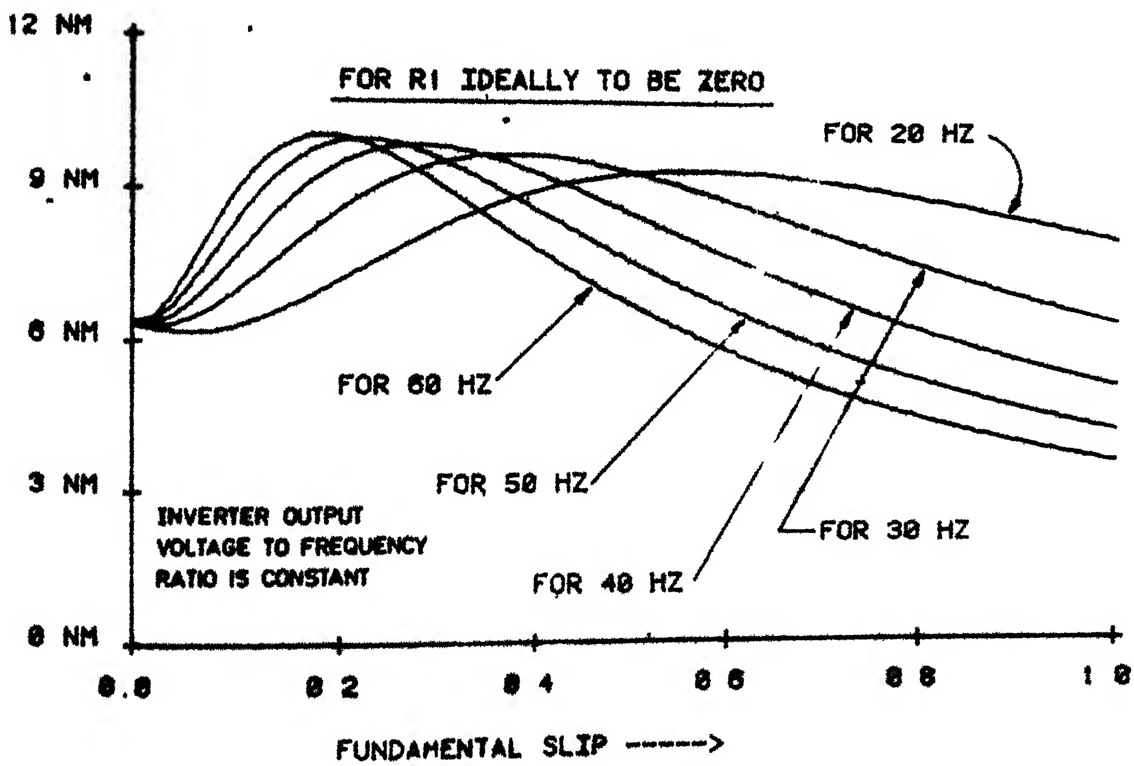


FIG 4.14(B)   SIXTH HARMONIC TORQUE SLIP CHARACTERISTICS

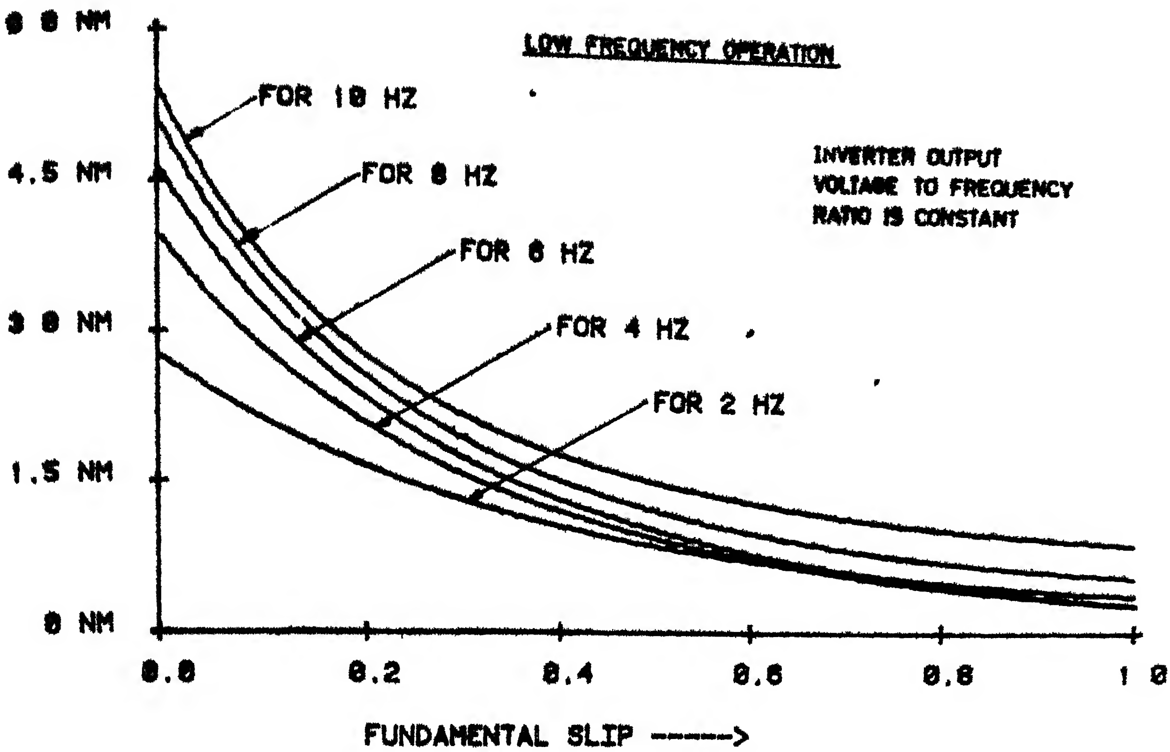


FIG 4.14(C) SIXTH HARMONIC TORQUE SLIP CHARACTERISTICS

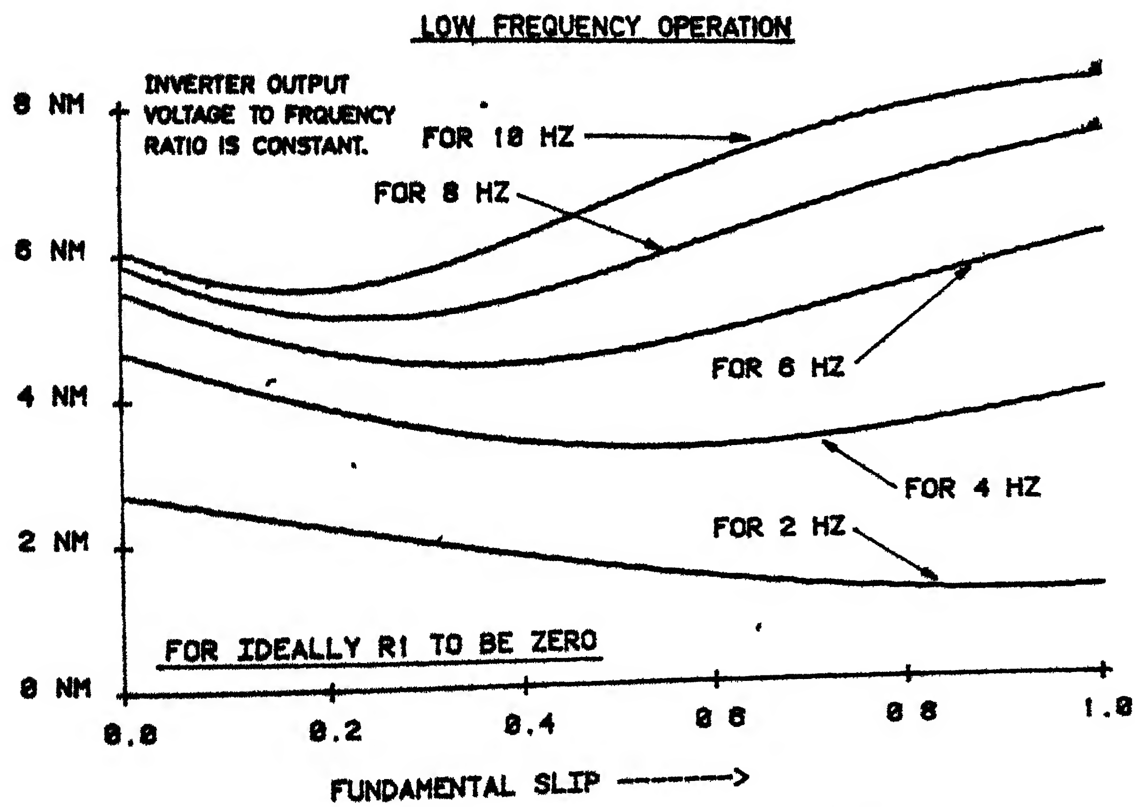


FIG 4.14(D) SIXTH HARMONIC TORQUE SLIP CHARACTERISTICS.

Chapter - 5

REDUCTION OF CURRENT AND TORQUE HARMONICS BY MODULATION OF INVERTER OUTPUT VOLTAGE

5.1 INTRODUCTION

Variable speed drives utilizing an a.c. motor fed with variable frequency, 3 phase a.c. derived from an inverter are encountered with increasing frequency. An inverter used for this purpose should, in addition, regulate the voltage at different inverter operating frequencies so as to maintain the flux density within the machine at the normal operating value. For modest speed ranges, a six step inverter with variable d.c. power supply can meet the requirement of low or very low speed operation. But the following problems become significant.

(i) Due to non-sinusoidal voltage waveforms, harmonic currents can lead to increase in motor heating and increased peak current . The harmonic currents are limited principally by the motor leakage inductances and are independent of the load.

(ii) The harmonics of the six step waveform interact with the fundamental to produce low frequency torque pulsations which may cause irregular rotation at very low speed.

(iii) The supply voltage becomes so low as to impair the commutation capability of the inverter.

To get rid of the first two problems mentioned, it is necessary to make the wave as nearly sinusoidal as possible. A filter between the inverter and the load can accomplish this task. But for large power , the filters become both bulky and costly. Besides, for variable frequency, the effectiveness of the LC filter is limited. Again, the simple step wave output from an inverter could be fed to a multiwinding transformer, so that variable voltage stepped waveform could be synthesized [19]. The greater the number of steps per cycle, the closer is the wave to a sinusoid, but still the equipment is large and costly.

Now the three problems mentioned above can be over come by the use of pulse width modulation. The pulse width modulated inverter has the distinctive feature that it obtains its input power from a d.c. source of constant magnitude which helps successful thyristor commutation. Pulse width modulation may be thought of as the control of the average level of a quantity (voltage, current, flux etc.) by applying or driving that quantity in discrete intervals or pulses (as shown in the typical inverter output waveform of Figures 5.3) depending upon the modulation techniques. These results in a fundamental output voltage of adjustable magnitude and frequency. The set of rules determining the sequence and timing of the thyristor gating is termed as the modulation policy. Affording both frequency and voltage control in one set of controlled semiconductor

device, it eliminates the need of multiple controlled power sections. This type of inverter is characterized in relation to other a.c. drives where it utilizes a reduced number of power semiconductors at the expense of more control circuitry and more finesse regarding the operation of the control circuit. The output voltage harmonics are raised to the vicinity of the inverter carrier frequency and the resulting high frequency torques are rendered harmless by the low pass mechanical characteristics. So, in brief, the practical benefits of the P.W.M. inverters are as follows:

(i) Simplified power circuitry

(ii) Common bus operation

(iii) Superior dynamic response over other inverters

(iv) Low and medium frequency operation

(v) Improved motor performance.

Therefore, the P.W.M. inverters may now be advantageously applied to provide precise motor performance.

The choice of a particular method or mode of pulse width modulation depends on the requirements of the application. It involves the balancing of the losses in the inverter incurred in using high switching frequency against the improved performance and the reduced losses in the motor.

Inverter producing six-stepped voltage waveform, the predominant pulsating torque is at the sixth harmonic

In section 5.3, pulse width modulation techniques have been discussed, however details of the control circuit have not been considered. Here only d.c. reference signal is considered to obtain equal pulse width waveforms.

Section 5.4 depicts the generated expression for Fourier components of any pulse width modulated waveform, in which the said waveform is taken as having only half wave symmetry.

In section 5.5 characteristics of equal pulsed waveform were considered. Detailed discussions have been made in section 5.5.1 on harmonic contents in the equal pulse width voltage waveforms. Nature of motor currents in the said modulated inverter fed induction motor has been discussed in section 5.5.2. In this regard only the effect of modulation index and number of pulses per half cycle on the fundamental and harmonic stator currents have been considered. Nature of electromagnetic torques have been discussed in section 5.5.3 for the induction motor under consideration. In this regard steady harmonics and pulsating harmonic torques were mainly considered. Here also effect of modulation index as well as number of pulses per half cycle on the above-mentioned torque is mentioned. Lastly in section 5.5.4, torque pulsations resulting from current harmonics are briefly discussed.

5.2 OPERATION OF A 3 PHASE PULSE WIDTH MODULATED INVERTER WITH 180° CONDUCTION

Although many modes of pulse width modulation may be devised and demonstrated, the basic operation of the inverter is simple, because of its simple switching action.

Figure 5.1(a) shows a simplified bridge inverter circuit and Figure 5.1(b) shows its analog composed of switches. In Figure 5.1(a), S_1, S_2, S_3....S_6 are the main thyristors and S_{1a}, S_{2a}, S_{3a},....S_{6a} are the auxiliary units which serve to turn off the main thyristors. D_1,D_2,..D_6 are the free wheeling diodes and C_a, L_a, C_b, L_b, C_c, L_c are the commutating components. For example, if S_1 is conducting, it may be turned off by firing thyristor S_{1a} which connects the charged capacitor C_a, thereby diverting the current from thyristor S_1. The difference of capacitor current and load current flows through the diode D_1, so that S_1 becomes reverse biased and turned off. Due to presence of load inductance, which keeps the current sensibly constant during commutation, load current continue to flow after the commutation of S_1. Similarly thyristors S_2 to S_6 are turned off by the action of the auxiliary thristors S_{2a}....S_{6a} respectively. In case of pulse width modulated voltage waveform, the conducting thyristors are retriggered and turned off several times depending upon the modulation technique during a particular mode period (one sixth of the inverter period). In P.W.M. inverter with 180° conduction

control technique the line to line voltages are completely defined independent of the load like unmodulated six step waveforms (Figure 2.4). Each line to line voltage can have one of three states, i.e. $+V_{dc}$, $-V_{dc}$ or zero. In case of nonzero load voltage, three thyristors conduct at a time, one from the top and two from the bottom or vice versa. But in case of zero load voltage in all phases (shown in section 2.4.1), three thyristors either all from the top or all from the bottom will have to conduct to provide a short circuit path of the load. This is achieved by turning off the only conducting thyristor from the top or bottom which was conducting earlier and triggering corresponding complementary thyristor.

As load terminals are always connected with the positive or negative bus, the theoretical line to d.c. neutral (v_{ao}, v_{bo}, v_{co}) voltages will always be $+V_{dc/2}$ or $-V_{dc/2}$. Example of modulated voltage waveforms are shown in Figure 5.3 along with the schedule of thyristor conduction.

As load voltages are defined at all periodic intervals, so Fourier analysis can be applied to these waveforms as they satisfy dirichlet conditions, irrespective of its complexity.

5.3 PULSE WIDTH MODULATION TECHNIQUES

Block diagram of the basic pulse width modulated inverter scheme is illustrated in Figure 5.2. Assuming that desired inverter frequency is to be set by means of a potentiometer, the voltage control oscillator produces a clock of constant amplitude, 50% duty cycle pulses. The frequency of the clock is six times the desired inverter output frequency. The output of the six state ring counter which acts in response to this square wave input and carrier comparator output distributes the switching commands to each of the main thyristors of the Figure 5.1(a) through proper gating logic. From the Figure 5.2 it is also seen that the setting of the frequency adjusting potentiometer also affects the generation of the other regulating signals called the "carrier" and the "voltage reference".

In Figure 5.3, voltage reference present at the output of V/Hz generator is taken as d.c. voltage. This voltage reference give rise to "equal pulse width" modulation. The amplitude of (A_r) of the reference signal is a measure of the required inverter fundamental output voltage. Controls are provided to allow generation of a linear inverter V/Hz characteristics. The output of the carrier generator is an isosceles traingle of fixed amplitude (A_c) whose frequency is an integral multiple ($R_{c/f}$ = 6,9,12...etc.) of desired inverter output frequency (f_o).

The triangular output from the carrier generator has been compared with the d.c. reference signal by means of the carrier comparator in Figure 5.3(b). The duty cycle of the comparator output is a measure of the required fundamental inverter output voltage. These signals have been used to modulate the output of the six state ring counter. Modulated ring counter outputs resulting from the modulation is shown in Figure 5.3(c).

Figure 5.3(d) shows the schedule of the gate pulses to the main thyristors of the three phase inverter power bridge [Figure 5.1(a)]. The resulting modulated theoretical line to d.c. neutral modulated line to line and modulated line to load neutral voltages are shown in Figure 5.3 for equal pulse width modulation.

5.4 GENERAL EXPRESSION FOR FOURIER COMPONENTS OF A PULSE WIDTH MODULATED WAVEFORM, $v(\omega_s t)$

A general analysis of the line to neutral voltage waveform (Figure 5.5) can be made as follows:

Any periodic waveform having zero average value, can be expressed by a Fourier series as

$$v(\omega_s t) = \sum_{n=1,2,3}^{\infty} c_n . \sin(n\omega_s t + \nu_n)$$

where, $c_n^2 = a_n^2 + b_n^2$; $\nu_n = \tan^{-1}(\frac{a_n}{b_n})$

and $a_n = \frac{1}{\pi} [\int_0^{2\pi} v(\omega_s t) . \cos(n\omega_s t) . d(\omega_s t)]$

$$b_n = \frac{1}{\pi} \left[\int_0^{2\pi} v(\omega_s t).\mathrm{Sin}(n\omega_s t).d(\omega_s t) \right]$$

and waveform having half wave symmetry i.e. $v(\omega_s t) = -v(\omega_s t+\pi)$ will contain only odd harmonics and depending upon odd or even function, the series contain only sine or only cosine terms respectively. But if the function is not even nor odd, then both sine and cosine terms will be present.

For p number of pulses per half cycle having different level of voltages as shown in Figure 5.5, we have the following equations (as per half wave symmetry)

$$a_n = \frac{2}{\pi} \left[V_1 \int_{\alpha_1}^{\beta_1} \mathrm{Cos}(n\omega_s t).d(\omega_s t) + V_2 \int_{\alpha_2}^{\beta_2} \mathrm{Cos}(n\omega_s t).d(\omega_s t) + \ldots + \ldots V_p \int_{\alpha_p}^{\beta_p} \mathrm{Cos}(n\omega_s t).d(\omega_s t) \right]$$

$$\text{or } a_n = \frac{2}{\pi n} \left[\sum_{k=1,2,3}^{p} V_k \, (\mathrm{Sin}\, n\beta_k - \mathrm{Sin}\, n\alpha_k) \right] \qquad (5.1)$$

$$\text{and similarly } b_n = \frac{2}{\pi n} \left[\sum_{k=1,2,3}^{p} V_k \, (\mathrm{Cos}\, n\alpha_k - \mathrm{Cos}\, n\beta_k) \right] \qquad (5.2)$$

where V_1, V_2,....V_k are the different level of voltages.

ω_s : angular output frequency of the inverter.

Equations (5.1) and (5.2) can also be written as

$$a_n = \frac{4}{\pi n} \left[\sum_{k=1,2,3}^{p} V_k \, \mathrm{Cos}\, \frac{n}{2}(\alpha_k + \beta_k).\mathrm{Sin}\, \frac{n}{2}(\beta_k - \alpha_k) \right] \qquad (5.3)$$

$$\text{and}\quad b_n = \frac{4}{\pi n}\left[\sum_{k=1,2,3}^{p} V_k . \mathrm{Sin}\,\frac{n}{2}(\alpha_k+\beta_k) . \mathrm{Sin}\,\frac{n}{2}(\beta_k-\alpha_k)\right] \tag{5.4}$$

Therefore, harmonic amplitude and harmonic phase angle can be determined by the following equation:

$$c_n = \frac{4\sqrt{2}}{\pi n}\Big[\{\Sigma V_k \mathrm{Cos}\,\frac{n}{2}(\alpha_k+\beta_k).\mathrm{Sin}\,\frac{n}{2}(\beta_k-\alpha_k)\}^2$$

$$+ \{\Sigma V_k \mathrm{Sin}\,\frac{n}{2}(\alpha_k+\beta_k).\mathrm{Sin}\,\frac{n}{2}(\beta_k-\alpha_k)\}^2\Big]^{1/2} \tag{5.5}$$

$$\text{and}\ \nu_n = \tan^{-1}\left\{\frac{\Sigma V_k \mathrm{Cos}\,\frac{n}{2}(\alpha_k+\beta_k).\mathrm{Sin}\,\frac{n}{2}(\beta_k-\alpha_k)}{\Sigma V_k \mathrm{Sin}\,\frac{n}{2}(\alpha_k+\beta_k).\mathrm{Sin}\,\frac{n}{2}(\beta_k-\alpha_k)}\right\} \tag{5.6}$$

$$\text{and}\ v(\omega_s t) = \Sigma c_n \mathrm{Sin}\,(n\omega_s t + \nu_n) \tag{5.7}$$

where $n = 1,2,3,\ldots\ldots p$.

So, by equating equation (5.3) or equation (5.4) to zero, we can eliminate one or more than one predominant harmonic [18]. Generally speaking, α_k is normally a variable controlling the level of the voltage, while β_k can be used to cancel harmonics. This technique can be used to extend the utility of harmonic cancelled systems without increasing the number of inverters. Certainly as more pulses are used, more harmonics can be eliminated. However, it also

becomes more and more difficult to generate the required inverter sequencing and switching losses also increase.

5.5 EQUAL PULSE WIDTH MODULATION

In this modulation, the traingular output from the carrier generator is compared with a constant d.c. voltage reference signal by means of the carrier comparator. The modulating output from the six state ring counter gives the proper pulse sequence to the three phase bridge inverter through gating logic. The voltage waveforms having equal pulse width are shown in Figures 5.3 and 5.4.

In this modulation, $(\beta_1 - \alpha_1) = (\beta_2 - \alpha_2) = \ldots = (\beta_p - \alpha_p)$

i.e. the widths of all the pulses are equal.

The general expression of the traingular carrier wave is

$$a_c(\omega_s t) = A_c \left[1 - \frac{2p}{\pi}\left\{\omega_s t - \frac{\pi}{p}(k-1)\right\}\right] \text{for } \frac{(k-1)\pi}{p} \leq \omega_s t \leq \frac{(2k-1)\pi}{2p}$$

– for negative slope (5.8)

and

$$a_c(\omega_s t) = A_c\left[\frac{2p}{\pi}\left\{\omega_s t - (k-1)\frac{\pi}{p}\right\} - 1\right] \text{for } \frac{(2k-1)\pi}{2p} \leq \omega_s t \leq \frac{\pi k}{p}$$

– for positive slope (5.9)

where A_c : the amplitude of the carrier wave

p : number of pulses/half cycle (i.e. $R_{c/f} = 2p$)

k : general pulse number.

Again equation for reference signal is $a_r(\omega_s t) = A_r$ (5.10)
where A_r : amplitude of the reference d.c. signal.

Defining modulation index (m_i) to be the ratio of amplitude of reference d.c. signal to the amplitude of traingular carrier wave, with the help of equations (5.8),(5.9) and (5.10) for the kth pulse,

$$\alpha_k \text{ (firing angle)} = (1-m_i)\frac{\pi}{2p} + \frac{\pi}{p}(k-1) \tag{5.11}$$

and

$$\beta_k \text{ (extinction angle)} = \alpha_k + \frac{\pi}{p} m_i \tag{5.12}$$

Now, the relation between firing and extinction angles are as follows:

$$(\alpha_1 + \beta_p) = (\alpha_2 + \beta_{p-1}) = \ldots = (\alpha_p + \beta_1) = \pi \tag{5.13}$$

From the equations (5.11) and (5.12), it is evident that for any number of pulses per half cycle, α_k of β_k will change linearly with the modulating index. Figure 5.6 shows the general firing and extinction angle characteristics for equal pulse width modulation. As number of pulses per half cycle increases, magnitude of the slope of the firing and extinction angle characteristics respectively decreases and increases proportionately.

5.5.1 Voltage Harmonics in Equal Pulse Width Modulated Three Phase Voltage Source Inverter

(a) Triplen Number of Pulses/Per Half Cycle

In case of triplen number of pulses per half cycle in the line to neutral voltage,

$$v(\omega_s t) = -v(\omega_s t + \pi) \text{ and } v(\omega_s t) = -v(2\pi - \omega_s t)$$

i.e. half wave symmetry and an odd function. Therefore, a_n of the equation (5.3) becomes zero and ν_n of the equation (5.6) becomes zero. So, with the help of equation (5.4) and the line to neutral voltage waveform of the Figure (5.3)

$$b_n = \frac{4V_{dc}}{3\pi n} \Big[\Big\{ \sum_{k=1,2,}^{q_1} \sin \frac{n}{2} (\alpha_k + \beta_k) . \sin \frac{n}{2} (\beta_k - \alpha_k) \Big\}$$

$$+ \Big\{ \sum_{k=q_1+1,q_1+2}^{q_2} 2 \sin \frac{n}{2} (\alpha_k + \beta_k) . \sin \frac{n}{2} (\beta_k - \alpha_k) \Big\}$$

$$+ \Big\{ \sum_{k=q_2+1,q_2+2}^{p} \sin \frac{n}{2} (\alpha_k + \beta_k) . \sin \frac{n}{2} (\beta_k - \alpha_k) \Big\} \Big] \quad (5.14)$$

where $q_1 = p/3$, $q_2 = 2p/3$,

p : triplen number of pulses/half cycle and

n : order of harmonic.

As v_{an}, v_{bn} and v_{cn} are balanced and lagging each other by $2\pi/3^c$, so only nontriplen odd harmonics will be present in the line to neutral voltages. Equation (5.14) can be used

to find the harmonic content in the line to neutral voltage for p = 3,6,9,12 etc., knowing corresponding α_k and β_k from equations (5.11) and (5.12).

A computer programme has been developed to find out the relative harmonic content in the inverter output voltages for any given pulse number.

Relative harmonic contents for equal pulse width modulated inverter output voltages are shown in Figure 5.7 (corresponds to Table 5.1) for triplen number of pulses per half cycle. The following points can be noted for the characteristics shown in the above-mentioned figure.

(i) As pulse width changes proportionately with the modulation index, the magnitude of the fundamental changes almost linearly with the modulation index (m_1) while the pulse position remains unchanged.

(ii) For constant modulation index, increase of pulse number does not affect the p.u. fundamental voltage significantly as the total pulse width for different levels of voltages remains constant.

(iii) As modulation index decreases from units (maximum fundamental voltage) to zero (zero fundamental output), those characteristics illustrate the rapid increase in relative low order harmonics specially at lower number of pulses per half cycle.

(iv) The harmonic peak, during the process of modulation yields high relative values for some values of m_1. For example the fifth harmonic at $m_1 = 0.5$, $p = 3$ is comparable to the fundamental [Figure 5.7(a)] .

(v) It is seen from Table 5.1 that fifth and seventh harmonics reduce as the number of pulses per half cycle is increased.

With triplen number of pulses 'p' per half cycle, only the $(2p \pm 1)$th order harmonics become predominant as compared with other harmonics.

The magnitude of harmonics lower than the predominant $(2p \pm 1)$th order, though small, do not change appreciably with the increasing pulse number.

(b) Non-triplen Number of Pulses Per Half Cycle

In case of non-triplen number of pulses per half cycle, the following features are present:

(i) The output line to line and line to neutral voltages have only half wave symmetry.

(ii) The instantaneous voltage functions are neither even nor odd, which results in existance of a_n, b_n and ν_n. And lastly,(iii) the line to line or line to neutral voltages become unbalanced. One example of such unbalancing is shown in Figure 5.4.

<u>For 4 pulses per half cycle</u>, with the help of equations (5.3) and (5.4) and the line to neutral voltages of the Figure 5.4, the following expressions for v_{an} can be written:

Case-I $\frac{1}{3} \leq m_i \leq 1$

$$a_n = \frac{4V_{dc}}{3\pi n}\Big[\cos\frac{n}{2}(\alpha_1+\beta_1)\sin\frac{n}{2}(\beta_1-\alpha_1)$$

$$+ \cos\frac{n}{2}(\alpha_2+\pi/3).\sin\frac{n}{2}(\pi/3-\alpha_2)$$

$$+ 2\cdot\cos\frac{n}{2}(\beta_2+\pi/3)\sin\frac{n}{2}(\beta_2-\pi/3) + 2.\cos\frac{n}{2}$$

$$(\alpha_3+2\pi/3).\sin\frac{n}{2}(2\pi/3-\alpha_3)$$

$$+ \cos\frac{n}{2}(2\pi/3+\beta_3).\sin\frac{n}{2}(\beta_3-2\pi/3) + \cos\frac{n}{2}$$

$$(\alpha_4+\beta_4).\sin\frac{n}{2}(\beta_4-\alpha_4)\Big] \qquad (5.15a)$$

and $$b_n = \frac{4V_{dc}}{3\pi n}\Big[\sin\frac{n}{2}(\alpha_1+\beta_1)\sin\frac{n}{2}(\beta_1-\alpha_1) + \sin\frac{n}{2}$$

$$(\alpha_2+\pi/3)\sin\frac{n}{2}(\pi/3-\alpha_2) + 2\cdot\sin\frac{n}{2}(\beta_2+\pi/3)$$

$$\sin\frac{n}{2}(\beta_2-\pi/3) + 2\sin\frac{n}{2}(\alpha_3+2\pi/3).\sin\frac{n}{2}(2\pi/3-\alpha_3)$$

$$+ \sin\frac{n}{2}(2\pi/3+\beta_3)\sin\frac{n}{2}(\beta_3-2\pi/3) + \sin\frac{n}{2}$$

$$(\alpha_4+\beta_4).\sin\frac{n}{2}(\beta_4-\alpha_4)\Big] \qquad (5.15b)$$

Case-II $\quad 0 < m_i < 1/3$

$$a_n = \frac{4V_{dc}}{3\pi n}\Big[\cos\frac{n}{2}(\alpha_1+\beta_1).\sin\frac{n}{2}(\beta_1-\alpha_1)+2.\cos\frac{n}{2}(\alpha_2+\beta_2).\sin\frac{n}{2}(\beta_2-\alpha_2)+2.\cos\frac{n}{2}(\alpha_3+\beta_3).\sin\frac{n}{2}(\beta_3-\alpha_3)+\cos\frac{n}{2}(\alpha_4+\beta_4).\sin\frac{n}{2}(\beta_4-\alpha_4)\Big] \tag{5.16a}$$

and

$$b_n = \frac{4V_{dc}}{3\pi n}\Big[\sin\frac{n}{2}(\alpha_1+\beta_1).\sin\frac{n}{2}(\beta_1-\alpha_1)+2.\sin\frac{n}{2}(\alpha_2+\beta_2).\sin\frac{n}{2}(\beta_2-\alpha_2)+2.\sin\frac{n}{2}(\alpha_3+\beta_3).\sin\frac{n}{2}(\beta_3-\alpha_3)+\sin\frac{n}{2}(\alpha_4+\beta_4).\sin\frac{n}{2}(\beta_4-\alpha_4)\Big] \tag{5.16b}$$

Therefore, c_n and ν_n can be easily computed from equations (5.15) and (5.16) for v_{an}. Similar expressions of harmonic amplitude and harmonic phase angle for v_{bn} and v_{cn} could also be written. Table 5.2 shows the relative harmonic content, all the phases for varying modulating index. It is observed that the magnitudes of the fundamental component of v_{an}, v_{bn} and v_{cn} for a constant modulation index are not equal, though total pulse widths in all the three phases are equal. Besides, harmonic contents are also not equal in all the three phases.

Here each line to neutral voltage changes its shape depending upon the operating range of the modulating

index whereas the total pulse width remains unchanged in all the three phases. The number of changes in the pattern of the above waveform depend on the non-triplen number of pulses per half cycle. It is also noticed that whenever α_k or β_k equals to a multiple of $\pi/3^c$ within a period of half cycle, the waveform changes beyond that values of α_k or β_k as m_i is changed. An example, to clear this complexity of the changes, can be given as follows.

From equations (5.11) and (5.12)

$$m_i = \left(2k - \frac{2p\alpha_k}{\pi} - 1\right) \qquad (5.17a)$$

and $$\alpha_k = \beta_k - \frac{\pi m_i}{p} \qquad (5.17b)$$

In case of $p = 4$, $\alpha_{1(max)}$ and $\beta_{1(max)}$ are less than $\pi/3$. From equations 5.17(a) and 5.17(b), $m_i = 1/3$ at $\alpha_2 = \pi/3$. Similarly $\beta_{2(max)}$ and $\alpha_{3(max)}$ are less than $2\pi/3$ and $m_i = 1/3$ for $\beta_3 = 2\pi/3$. So, waveshape will change beyond $m_i = 1/3$. That is why two cases have been considered while writing the Fourier coefficients of line to neutral voltage for $p = 4$.

Considering this unbalancing of the line as well as phase voltages shown in Table 5.2, non-triplen number of pulses per half cycle are not acceptable within the control

number of pulses are considered in later part of this Chapter and comparison has been made among different triplen number of pulses per half cycle.

5.5.2 Nature of Stator Current in Equal Pulse Width Modulated Voltage Source Inverter Fed Induction Motor

(a) Effect of Modulation, Index and Number of Pulses on r.m.s. Value of Fundamental Stator Current

As p.u. fundamental voltages for any constant modulation index are almost equal for different values of p, the pulses per half cycle (shown in Table 5.1), the r.m.s. value of the fundamental stator current for any given speed will change according to those p.u. values.

It is found from Figure 5.8(a) that at constant inverter frequency and motor speed, fundamental component of motor current changes almost linearly with the modulation index for any triplen number of pulses per half cycle.

But to maintain constant flux in the machine, fundamental component of motor current decreases slowly as modulation index decreases for any given motor speed. Figure 5.8(b) shows such typical characteristics though it is basically regulated by motor parameters and operating frequency of the inverter. Constant current characteristics would have been obtained if stator resistance and rotor time constant of the motor would have been made ideally zero and infinite respectively.

(b) Effect of Modulation Index and Number of Pulses on r.m.s. Values of the Harmonic Stator Current

Figures 5.9(a) and 5.9(c) show the example of the 5th and 7th harmonic current characteristics with varying modulation index for different number of pulses per half cycle at 60 Hz. It is observed that at constant frequency, harmonic currents change according to the change of their respective harmonic voltages corresponds to Figure 5.7.

The same characteristics are shown for constant voltage to frequency ratio in Figures 5.9(b) and 5.9(d) where it is noticed that at low values of modulation index, harmonic currents become highly comparable with the r.m.s. value of fundamental stator current.

Speed of the motor does not affect these harmonic currents for any particular value of modulation index and fixed number of pulses per half cycle. One such typical constant current characteristics is shown in Figure 5.9(e).

So, far only the effect of individual harmonic current is considered. But the combined effect due to all existing harmonics in the waveform may become sufficient enough to cause subsequent increase in motor amperes and heating. To investigate it, from equation (3.13)

$$i_{1q} = i_{11q} + i_{15q} + i_{17q} + \ldots\ldots \infty$$

$$= i_{11q} + i_{1hq}$$

(b) Effect of Modulation Index and Number of Pulses on r.m.s. Values of the Harmonic Stator Current

Figures 5.9(a) and 5.9(c) show the example of the 5th and 7th harmonic current characteristics with varying modulation index for different number of pulses per half cycle at 60 Hz. It is observed that at constant frequency, harmonic currents change according to the change of their respective harmonic voltages corresponds to Figure 5.7.

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$$i_{1q} = i_{11q} + i_{15q} + i_{17q} + \dots\dots \infty$$

$$= i_{11q} + i_{1hq}$$

where i_{1nq} is the instantaneous harmonic current. Therefore,

$$I_{1q}^2 = I_{11q}^2 + I_{1hq}^2 \text{ (in r.m.s. quantities)} \tag{5.18}$$

where $$I_{1hq}^2 = \sum_{n=5,7,11}^{\infty} I_{1nq}^2 \text{ or } \left(\frac{I_{1hq}}{I_{11q}}\right) = \sum_{n=5,7}^{\infty} \left\{\left(\frac{I_{1nq}}{I_{11q}}\right)^2\right\}^{1/2} \tag{5.19}$$

This ratio of r.m.s. value of harmonic current to r.m.s. value of fundamental current can be called as "current harmonic factor".

Typical current harmonic factor versus modulation index characteristics for different pulse number has been shown in Figure 5.10. The following points can be noted from the above figure.

(i) As at very low output voltages, low order harmonics are as large as fundamental output, as a result of which current harmonic factor increases significantly at low values of modulating index.

(ii) Increase of number of pulses per half cycle greatly affect this characteristics. Depending upon the relative harmonics contents, current harmonic factor generally decreases with increasing pulse number.

Table 5.3 shows the exact variation of this current harmonic factor which is, of course, a measure of total harmonic content in motor current with varying modulating index, motor speed and number of pulses per half

cycle.

Figure 5.11 depicts the example of instantaneous stator current waveforms for equal pulse width modulated voltage source inverter fed induction motor. Different number of pulses per half cycle and varying modulation index at constant inverter operating frequency are considered in the said figure. Here the peak current and the wave-shape for a particular motor parameters are regulated by the following factors

(i) r.m.s. value of fundamental stator current. This is mainly affected by motor speed, modulation index and inverter operating frequency.

(ii) Ratios of r.m.s. value of individual harmonic current to r.m.s. value of fundamental current. This is mainly affected by motor speed, modulation index, inverter operating frequency, number of pulses per half cycle and the order of harmonic.

(iii) Fundamental and harmonic power factor angles. These are mainly affected by inverter operating frequency and speed of the motor though the later one also depends on the order of harmonic.

5.5.3 Nature of Electromagnetic Torque in Equal Pulse Width Modulated Voltage Source Inverter Fed Induction Motor.

Effect of number of pulses per half cycle and modulation index on a steady harmonic or average torque and pulsating harmonic torque are studied in this section. A computer programme is also developed to calculate the above-mentioned quantities for any modulation index, number of pulses per half cycle and inverter operating frequency. Voltages having values lower than 0.1% of the fundamental are neglected in calculating the torque.

(a) Steady Harmonic Torque or Average Torque

Average torque is generally produced by the reaction of harmonic air gap fluxes with harmonic rotor currents of same order. Figure 5.12(a) shows the average torque-slip characteristics for varying modulating index at constant frequency of 60 Hz. This characteristics will be nearly same for any triplen number of pulses per half cycle.

Figures 5.12(b) and 5.12(c) show the variation of steady fifth and seventh harmonic torque with the modulation index for different number of pulses per half cycle at inverter operating frequency of 60 Hz. But these characteristics resemble the harmonic current versus modulation index characteristics of the Figures 5.9(a) and 5.9(c) because individual harmonic is only responsible to produce respective

steady harmonic torque . The same characteristics could have been drawn for constant flux in the machine, but in that case also it would resemble with the Figures 5.9(b) and 5.9(d).

(b) Pulsating Harmonic Torque

The same computer programme has been developed to find out all the possible pulsating harmonic torques taking all possible combination of harmonic rotating fluxes and harmonic rotor currents till the thirty seventh harmonic. Generally, principal pulsatory harmonic torques (sixth, twelfth, eighteenth etc.) arise from the interaction between fundamental rotating flux and harmonic rotor current and vice versa.

As sixth or sometime twelfth harmonic torque depending upon number of pulses per half cycle predominates over other pulsating harmonic torques, so only these two lowest order harmonic torques are considered in this sub-section.

Figure 5.13 shows the sixth harmonic torque versus modulation index characteristics for different motor speeds and number of pulses per half cycle. The following features can be noted from the said figure.

(i) In general, as speed of the motor increases pulsating sixth harmonic torque increases upto a certain speed of the motor. However, beyond this value of speed the

above torque may decrease with increase in speed.

(ii) The peak of this characteristics is mainly dependent on the relative lower order predominant harmonic contents in voltage. As fifth and seventh harmonics are highly comparable with the fundamental for three pulses per half cycle within a range of modulation index ($0.55 \leq m_1 \leq 0.75$), that is why sixth harmonic torque reaches its peak within the said range of modulation index irrespective of motor speed. This feature is shown in Figure 5.13(a).

Whereas for other higher triplen number of pulses per half cycle, sixth harmonic torque decreases with the modulation index.

(iii) As lower order harmonics get reduced with increase of number of pulses, the above torque decreases accordingly with the higher values of p.

Figure 5.14 shows the twelfth harmonic torque versus modulation index characteristics for different motor speeds. Here also the following points are notable.

(i) Increasing motor speed increases twelfth harmonic torque upto a certain speed as in the case of sixth harmonic torque stated earlier.

(ii) Though twelfth harmonic torque decreases with modulation index in general, but in case of 6 pulses per half cycle [Figure 5.14(b)], the characteristics resembles to the sixth harmonic torque versus modulation index characteristics for 3 pulses per half cycle [Figure 5.13(a)] with ≅ 50% reduction in magnitude.

In general, for 3p number of pulses per half cycle, this type of characteristics would occur for (6p)th pulsating harmonic torque,. So higher order pulsating harmonic torque can be greater than lower order pulsating harmonic torques depending upon the number of pulses per half cycle. As example, for 6 pulses per half cycle, Figure 5.13(b) and Figure 5.14(b) corresponds to sixth and twelth harmonic torques respectively.

Tables 5.4(a) and 5.4(b) show the sixth and twelfth harmonic torque respectively for different number of pulses per half cycle and motor speed, keeping modulated output voltage to operating frequency of the inverter constant.

5.5.4 Torque Pulsation in Induction Machine Driven by Equal Pulse Width Modulated Inverter

Inspection of Figure 5.7 reveals that the amplitudes of voltages at (2p-1)th and (2p+1)th frequency are very nearly same. This gives rise to large currents at these frequencies and hence large torque pulsation at (2p)th frequency. Figure 5.15 shows the instantaneous harmonic

torque (i.e. instantaneous electromagnetic torque - steady harmonic torque) computed for the machine under consideration. Figure 5.15(a) and Figure 5.15(b) indicate torque pulsations at six times the inverter operating frequency for 3 pulses per half cycle, whereas for 6 pulses per half cycle, the said pulsation occur at twelfth time the inverter operating frequency which are shown in Figure 5.15(c) and Figure 5.15(d).

So use of P.W.M. inverter in the mode of constant number of pulses per half cycle produces large torque pulsations as large voltages are available at harmonic frequencies adjacent to (2p) frequency. Generally low order predominant harmonics (2p-1)th and (2p+1)th are added to produce torque pulsations at (2p)th harmonic frequency. However, if number of pulses are large, these pulsations will be less due to the reduction in the harmonic currents. Besides, torque pulsations at high frequencies can be easily overcome by the mechanical inertia of the rotor shaft of the induction motor.

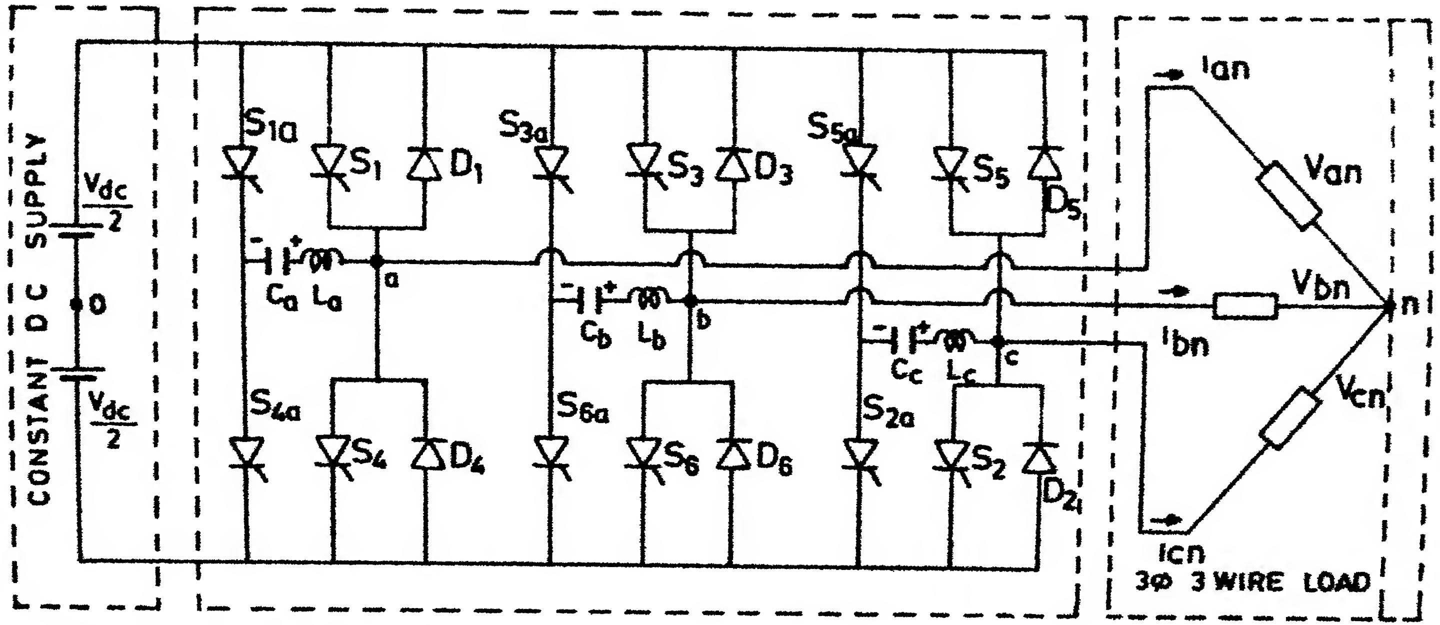


FIG 51(A) 3Φ INVERTER (auxiliary-commutated) FEEDING 3Φ LOAD.

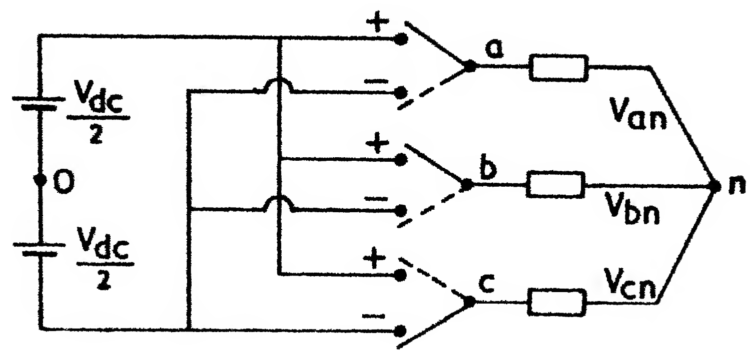
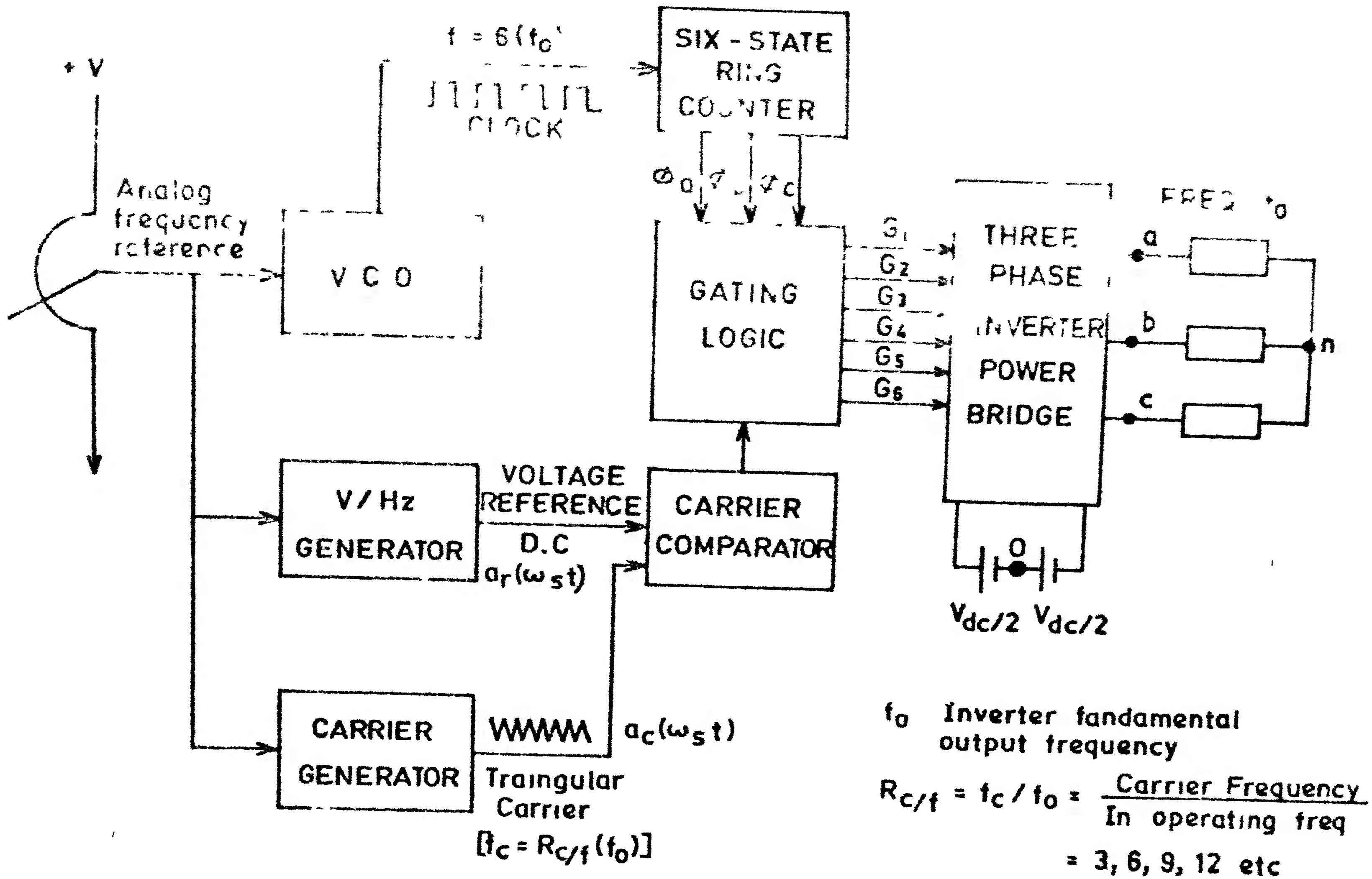


FIG 51(B) SWITCH ANALOG OF FIG 51(A)



f_o Inverter fandamental output frequency

$R_{c/f} = f_c / f_o = \frac{\text{Carrier Frequency}}{\text{In operating freq}}$

$= 3, 6, 9, 12$ etc

FIG 5 2 SIMPLE PULSE WIDTH MODULATED VOLTAGE SOURCE INVERTER SCHEME

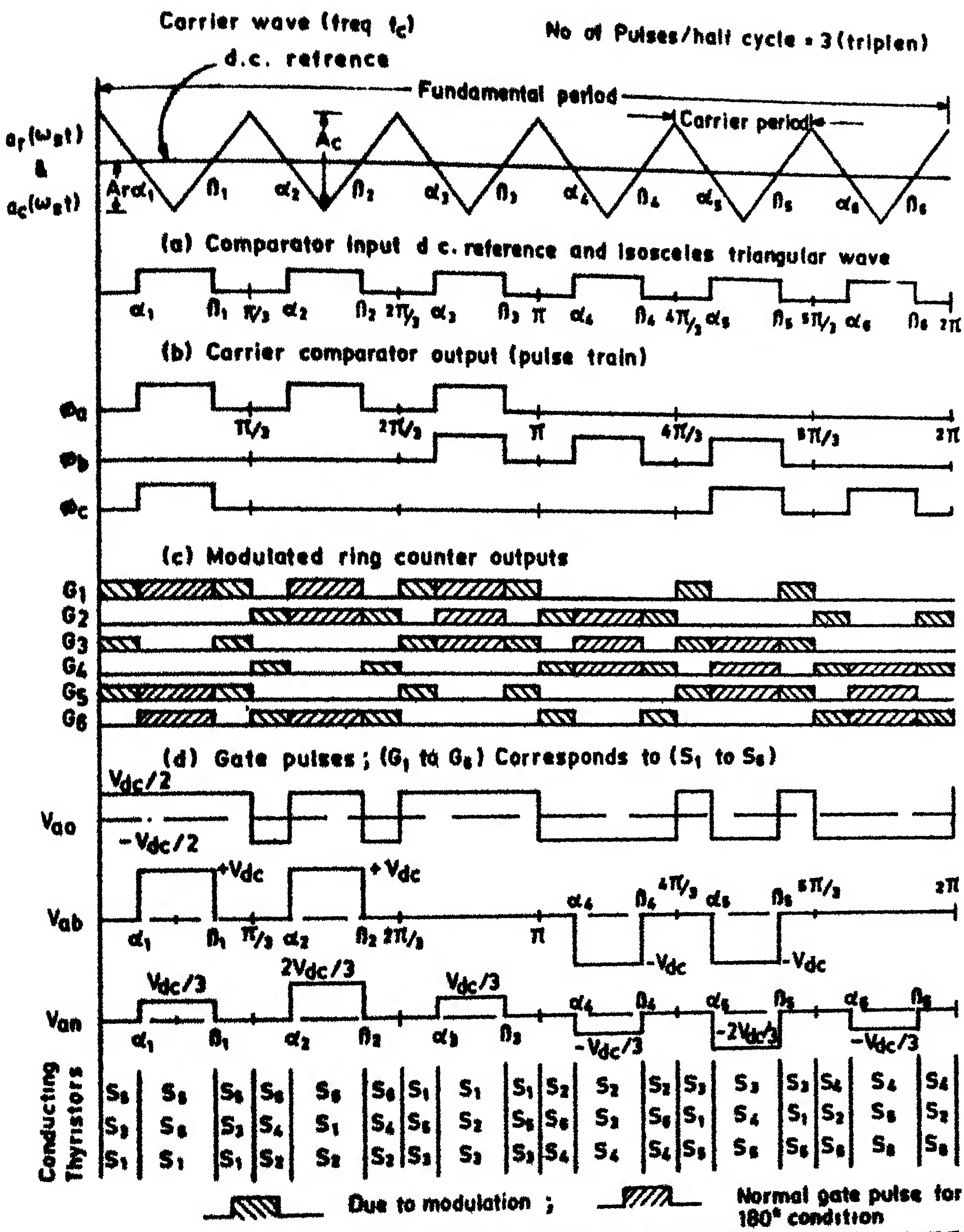


FIG. 5.3 WAVEFORMS OF EQUAL PULSE WIDTH MODULATED INVERTER OF FIG. 5.1(A) FOR $R_{c/f} = 6$.

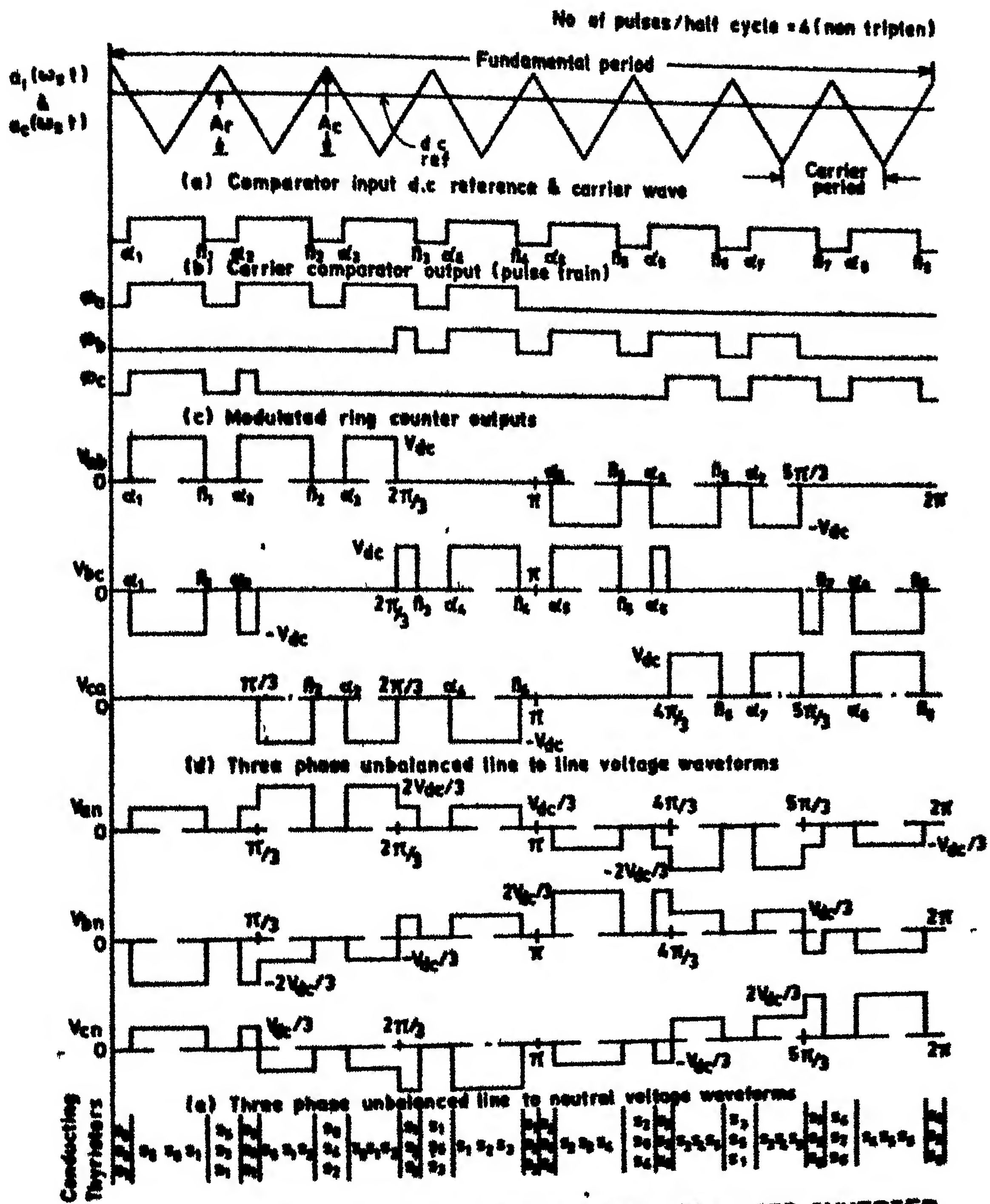


FIG. 5.4 WAVEFORMS OF EQUAL PULSE WIDTH MODULATED INVERTER.
(FOR $R_{C/R} = 8$)

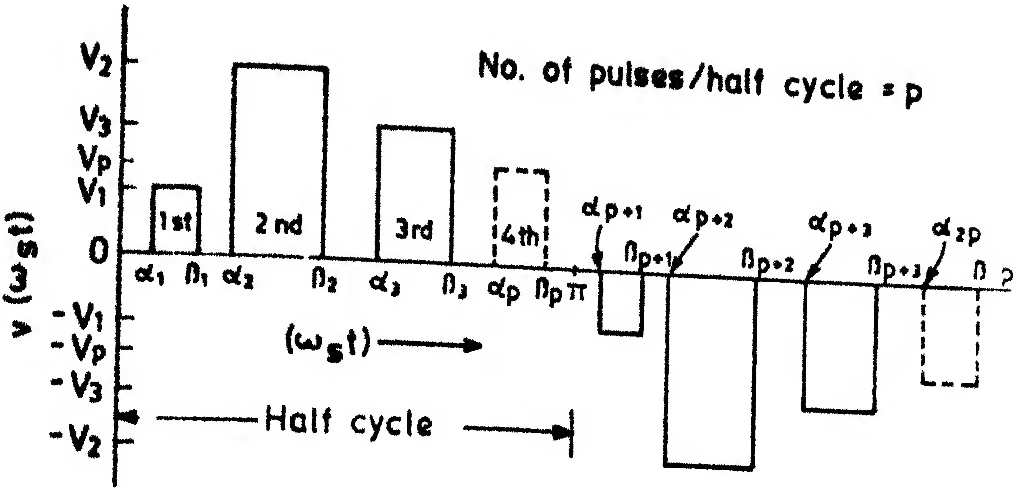


FIG 5 5 GENERAL PULSE WIDTH MODULATED VOLTAGE WAVEFORM

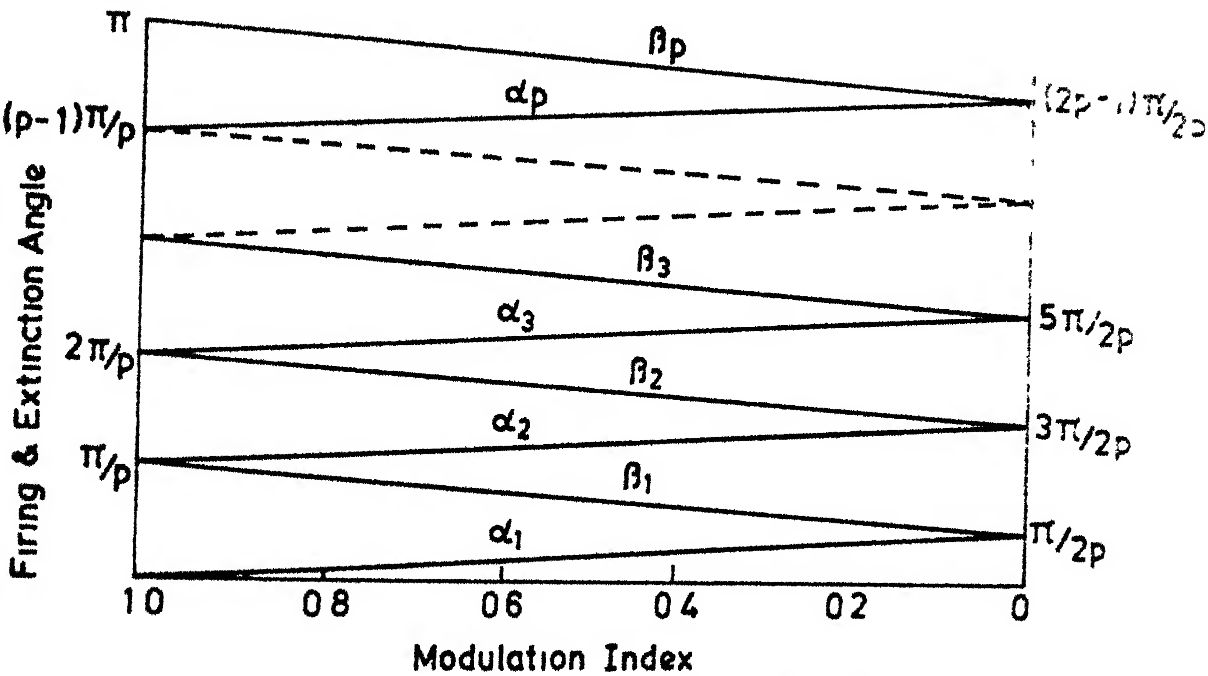


FIG 5 6 FIRING & EXTINCTION ANGLE CHARACTERISTICS FOR EQUAL PULSE WIDTH MODULATION

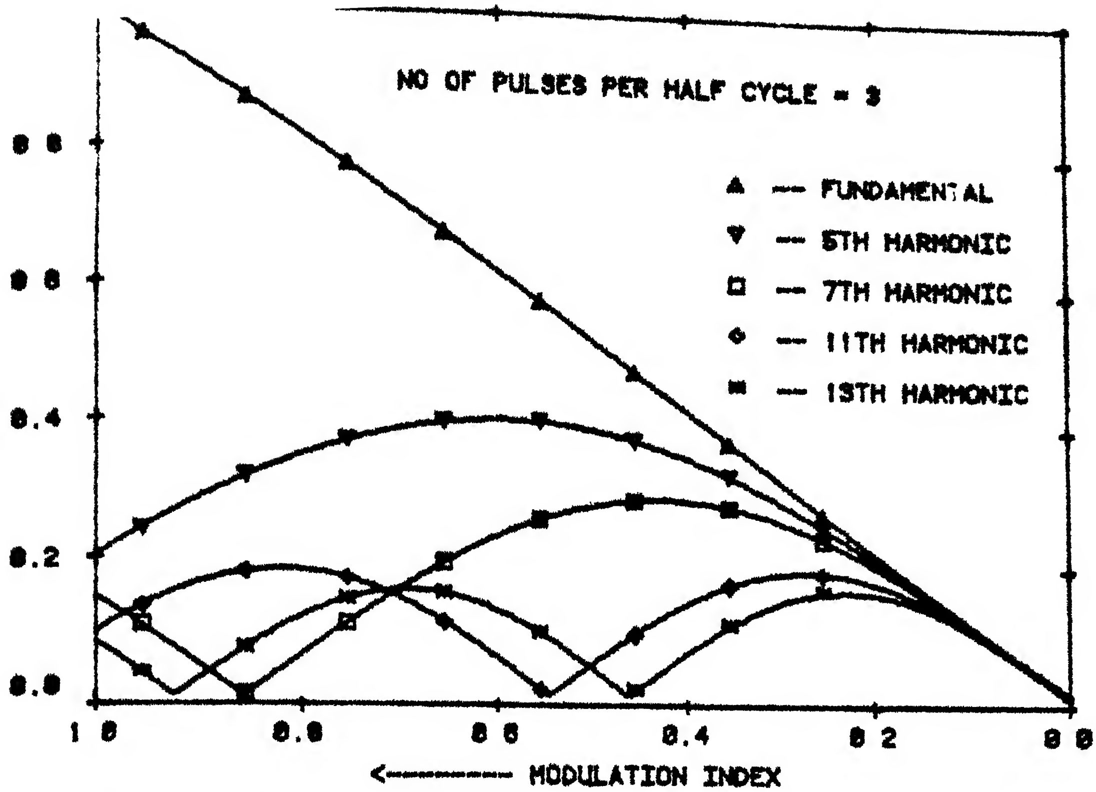


FIG 5.7(A) P.U. HARMONIC CONTENT IN VOLTAGE FOR EPWM VOLTAGE SOURCE INVERTER WITH VARYING MODULATION INDEX

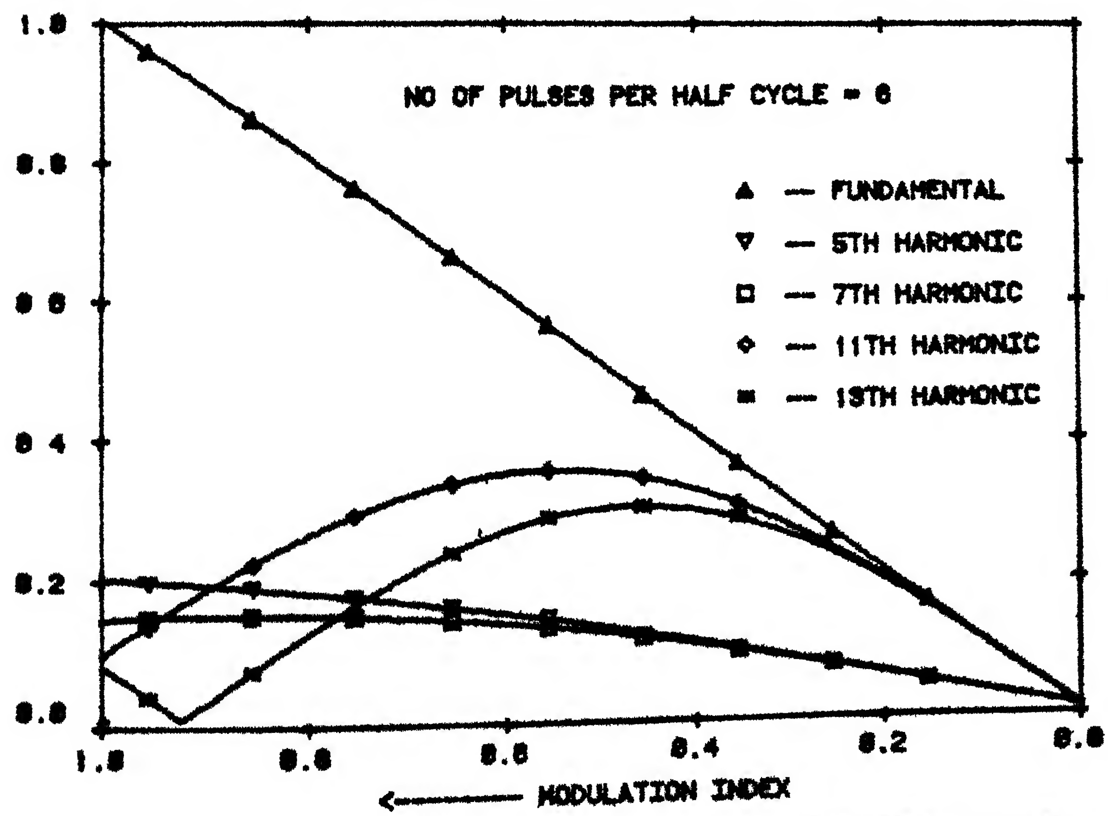


FIG 5.7(B) P.U. HARMONIC CONTENT IN VOLTAGE FOR EPWM VOLTAGE SOURCE INVERTER WITH VARYING MODULATION INDEX

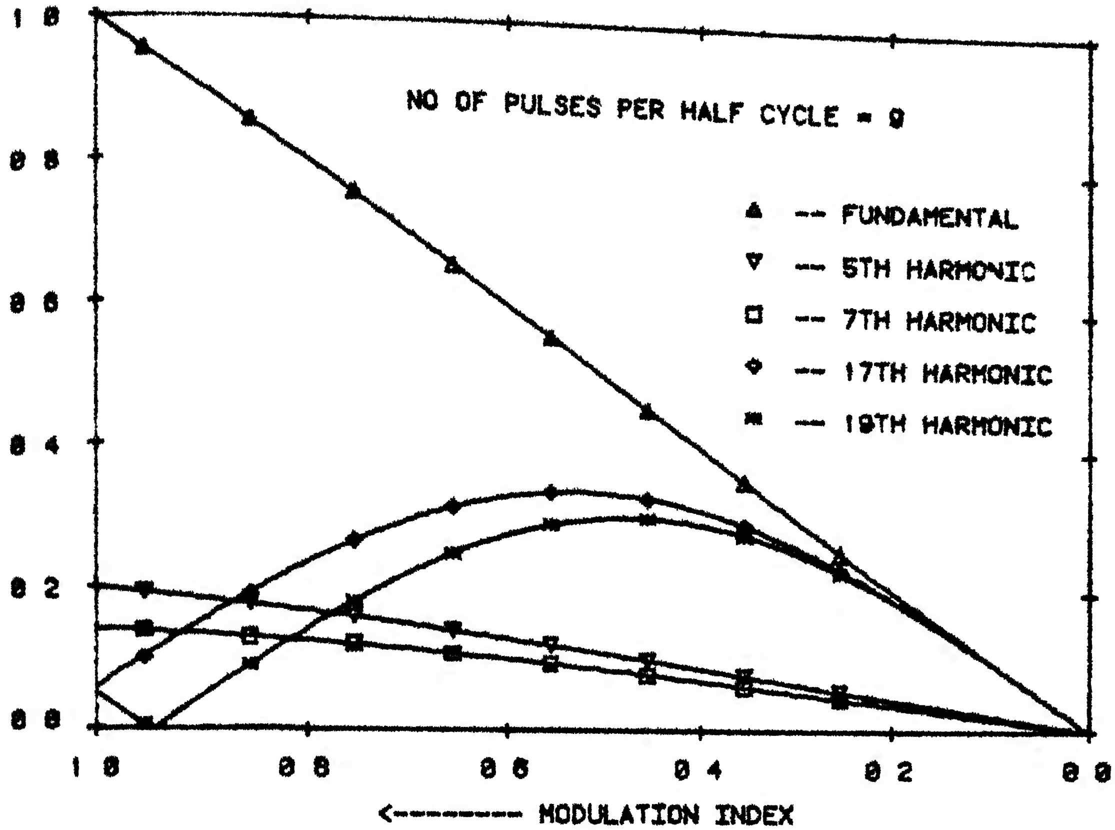


FIG 5 7(C) P U HARMONIC CONTENT IN VOLTAGE FOR EPWM VOLTAGE SOURCE INVERTER WITH VARYING MODULATION INDEX

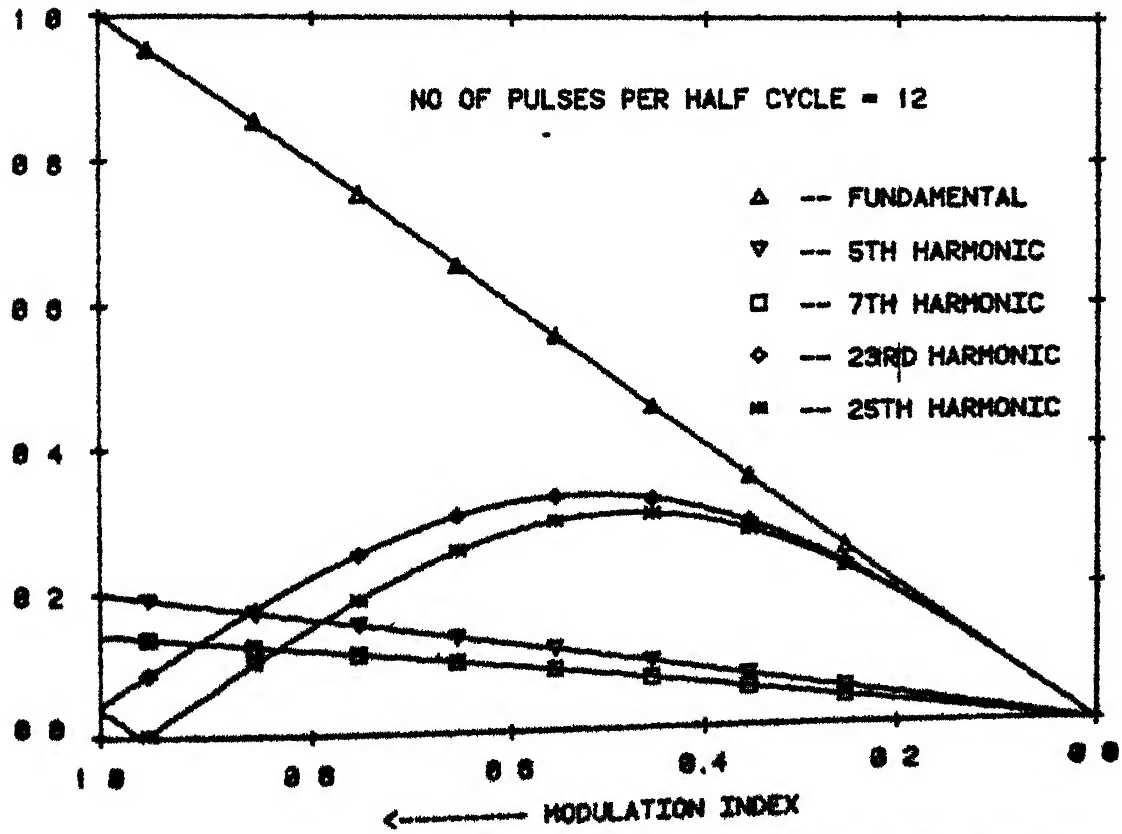


FIG 5 7(D) P U HARMONIC CONTENT IN VOLTAGE FOR EPWM VOLTAGE SOURCE INVERTER WITH VARYING MODULATION INDEX

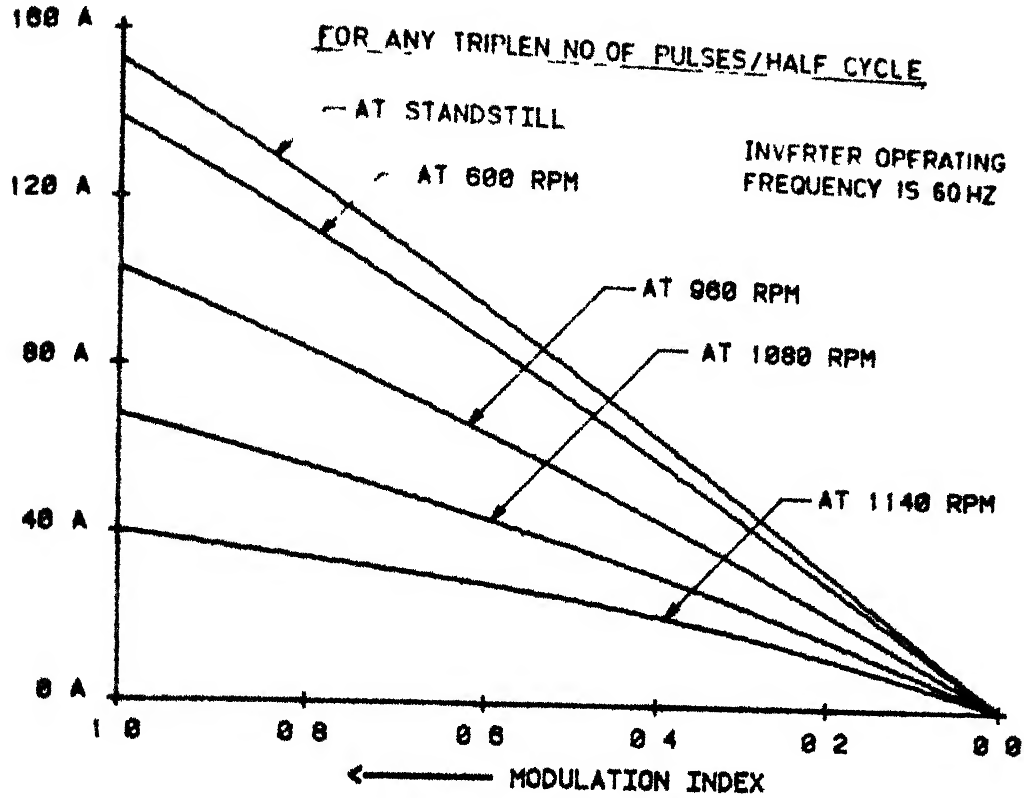


FIG 5 8(A) FUND. R M S. CURRENT VS. MOD INDEX CHARACTERISTICS

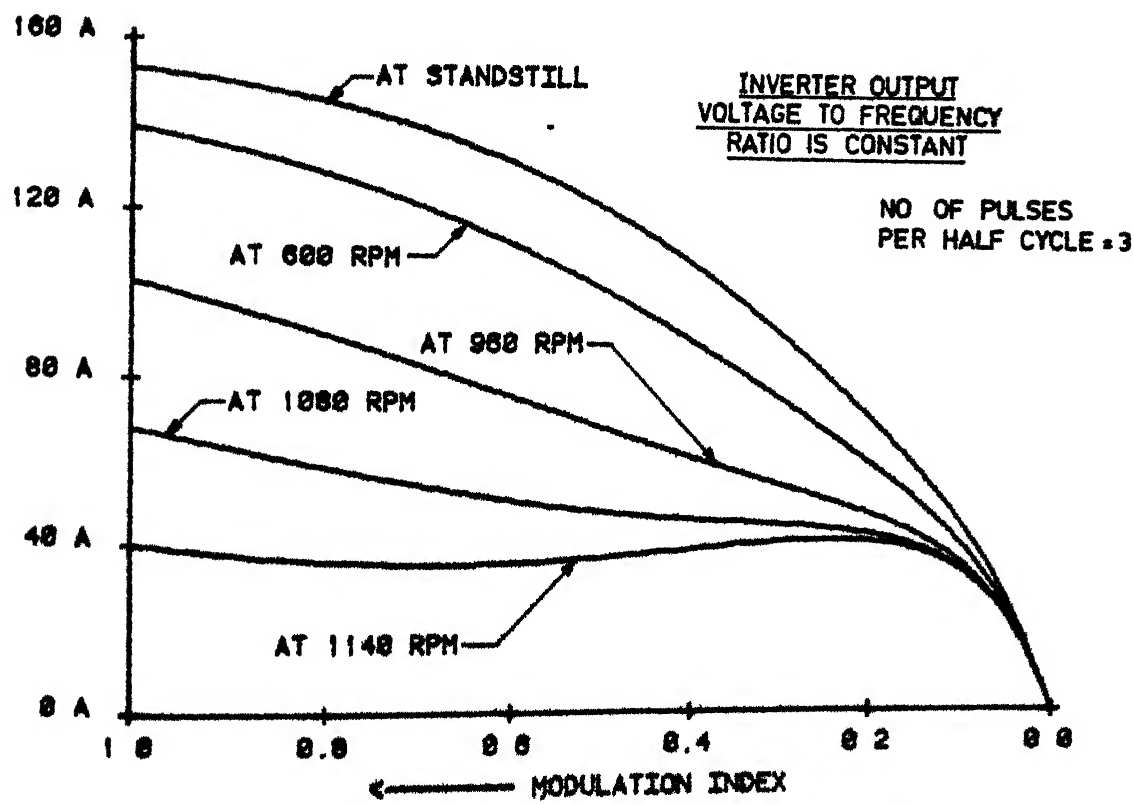


FIG 5 8(B) TYPICAL FUND. RMS CURRENT VS. MOD. INDEX CHARACTERISTICS.

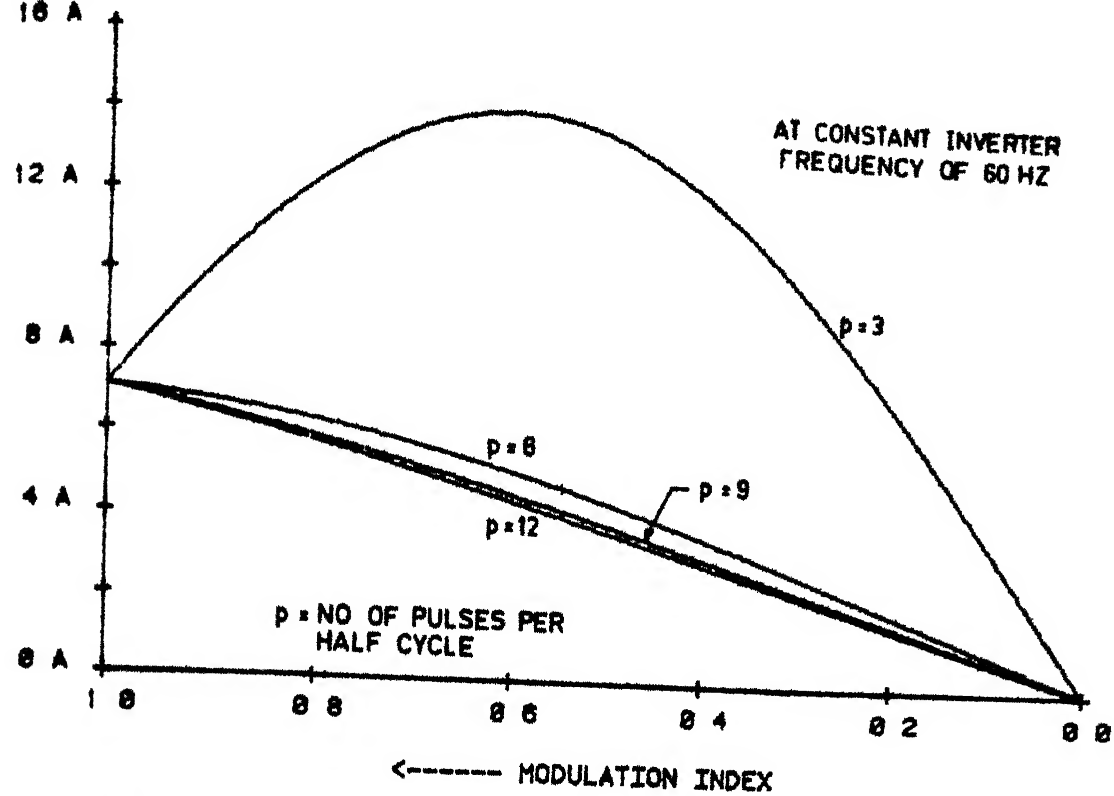


FIG 5 9(A) 5TH HAR R M S CURRENT VS MOD INDEX CHARACTERISTICS

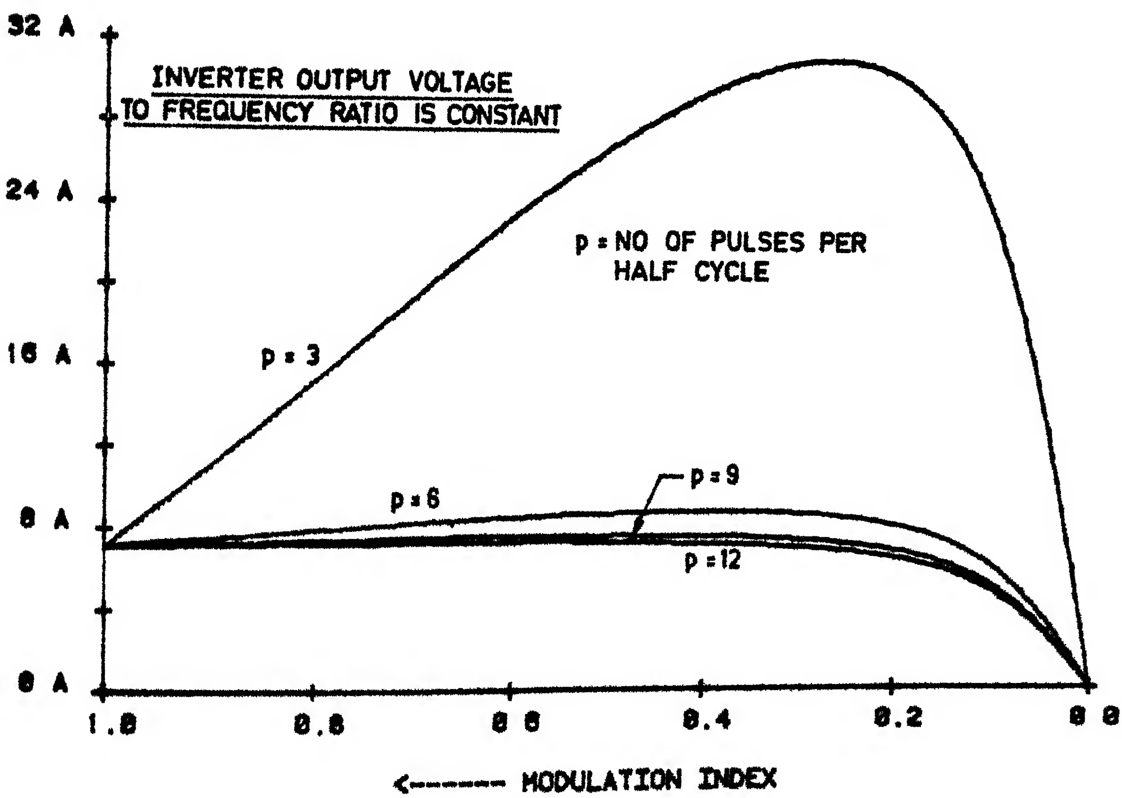


FIG 5 9(B) 5TH HAR R.M S. CURRENT VS. MOD INDEX CHARACTERISTICS

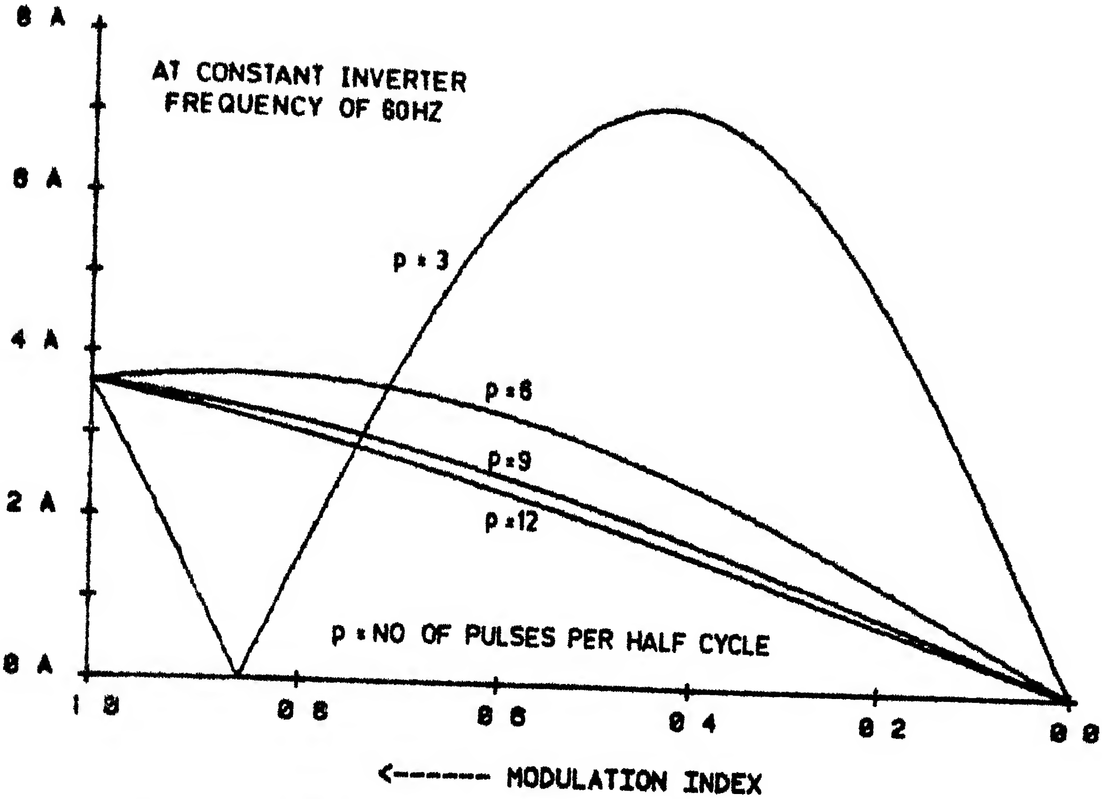


FIG 5 9(C) 7TH HAR R M S CURRENT VS MOD INDEX CHARACTERISTICS

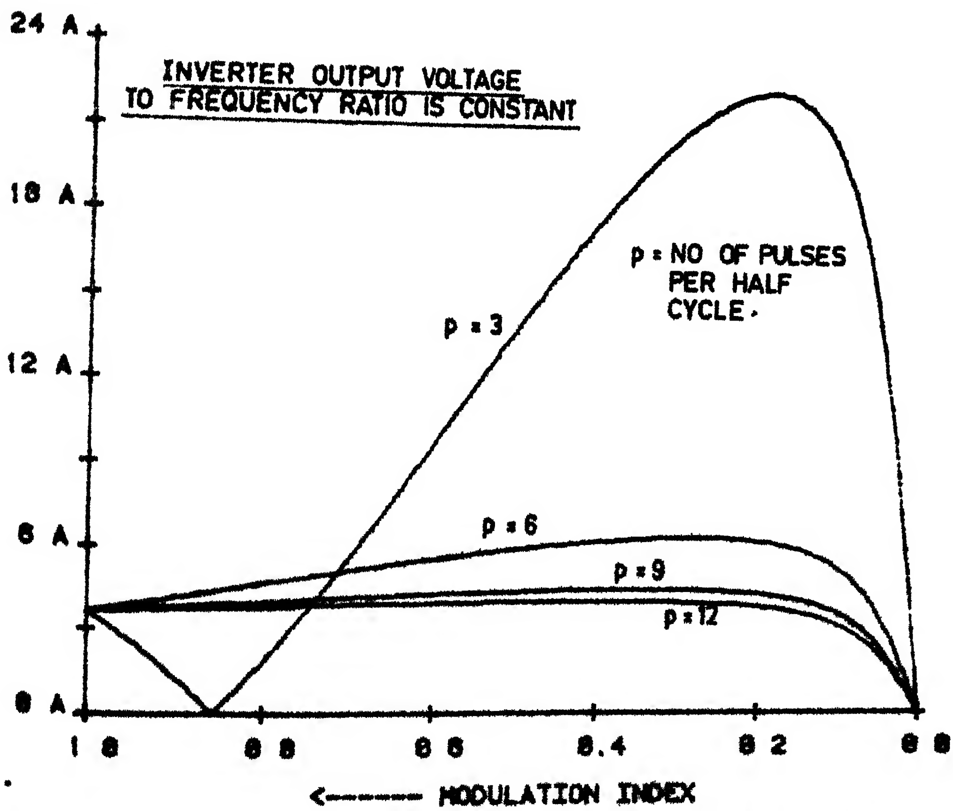


FIG 5 9(D) 7TH HAR R M S CURRENT VS. MOD INDEX CHARACTERISTICS

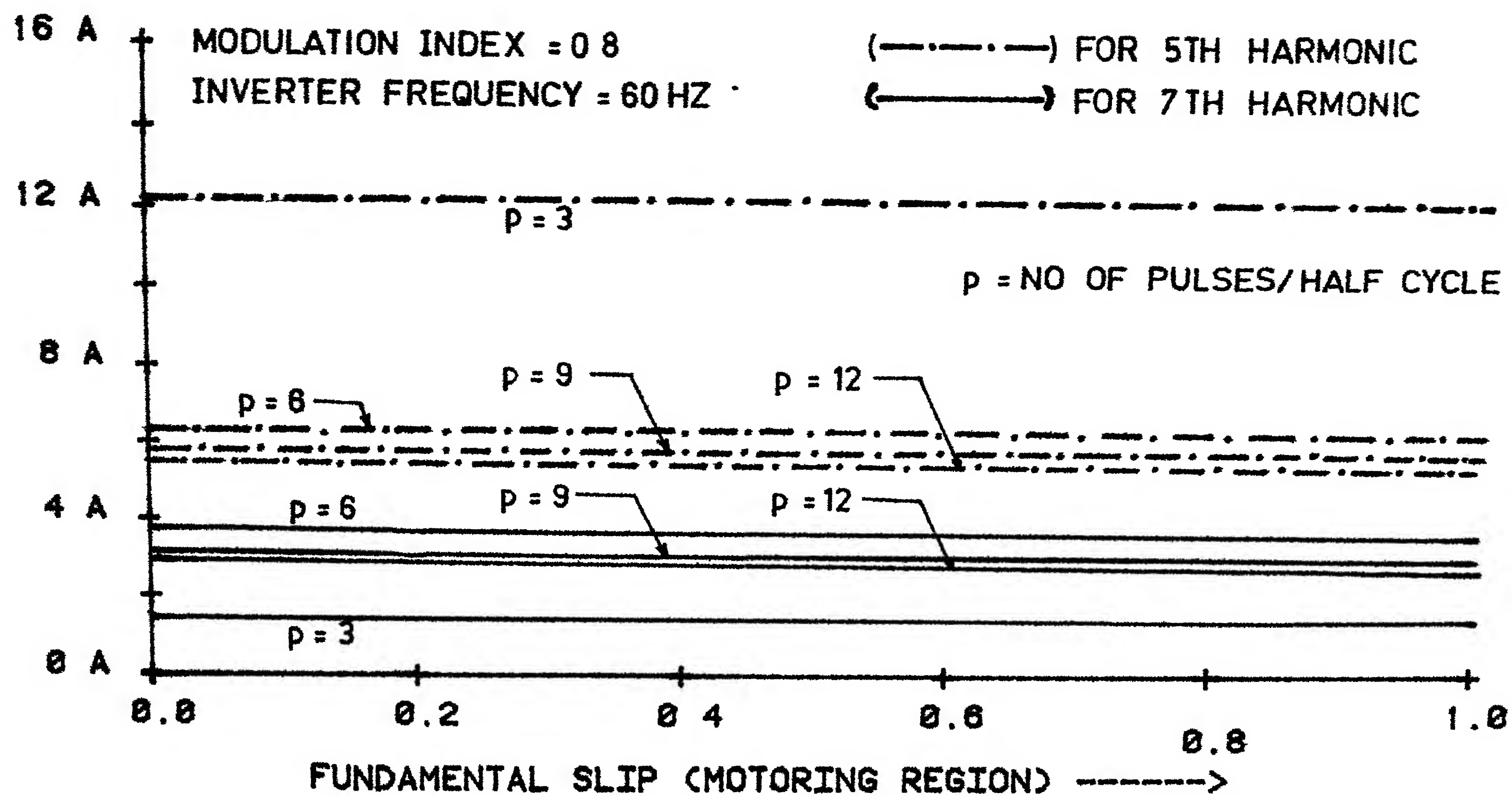


FIG 5.9(E) HAR R.M.S. CURRENT VS. FUND. SLIP CHARACTERISTICS.

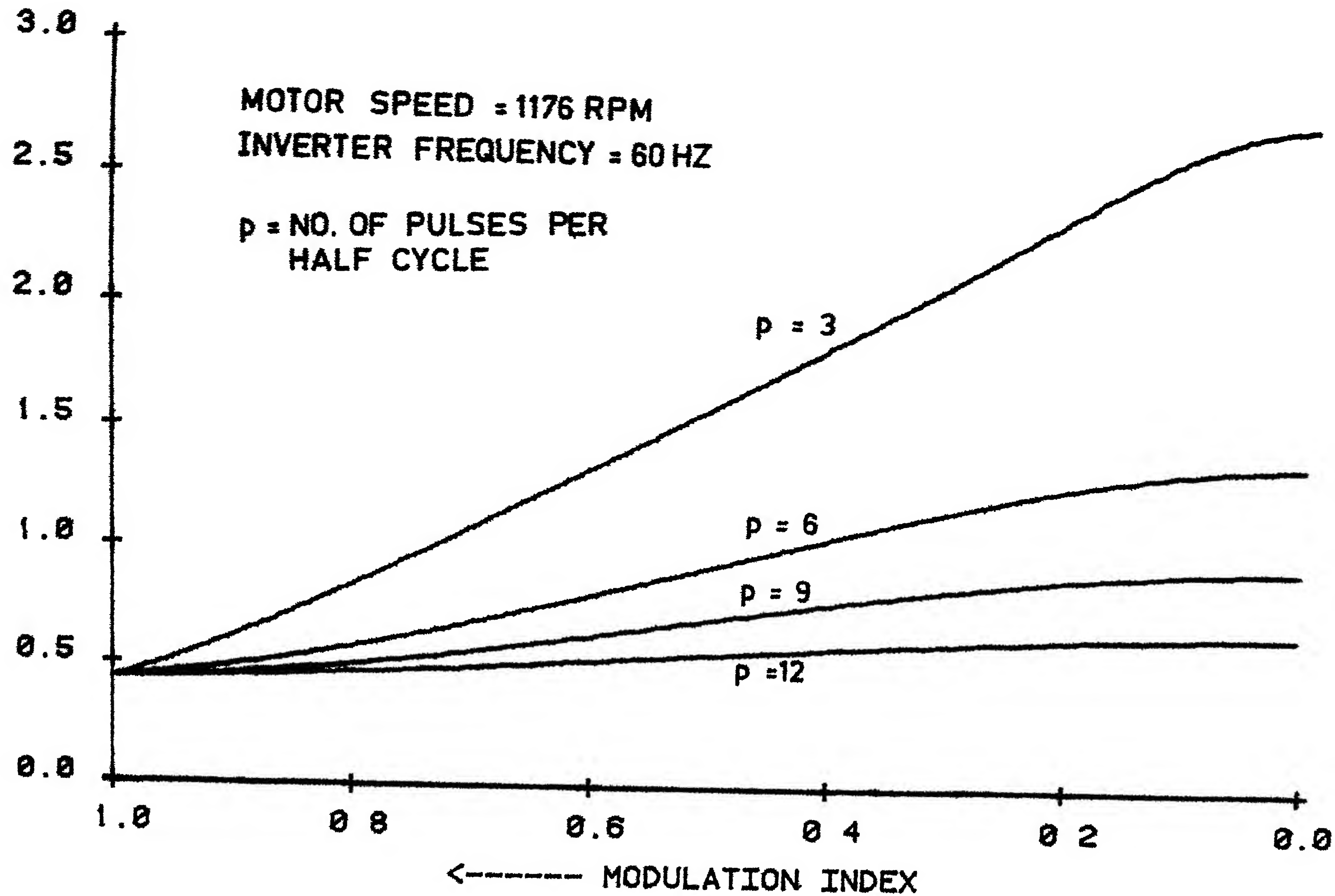
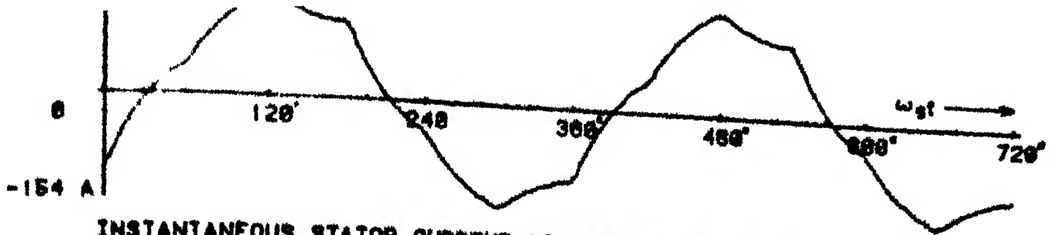
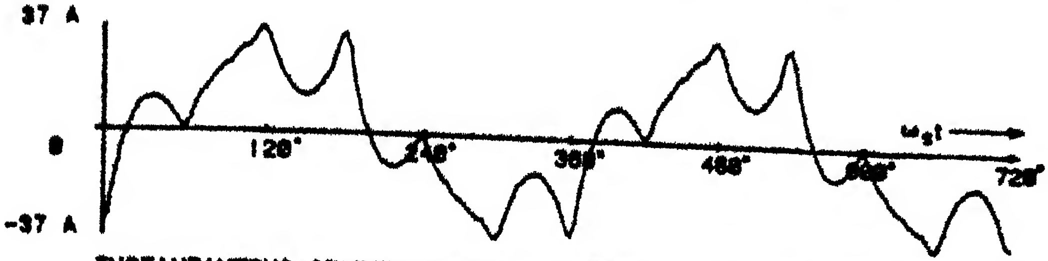


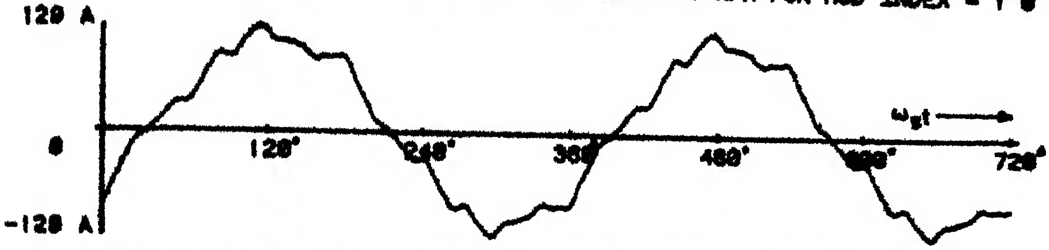
FIG 5.10 TYPICAL CURRENT HAR FACTOR VS. MOD INDEX CHARACTERISTICS.



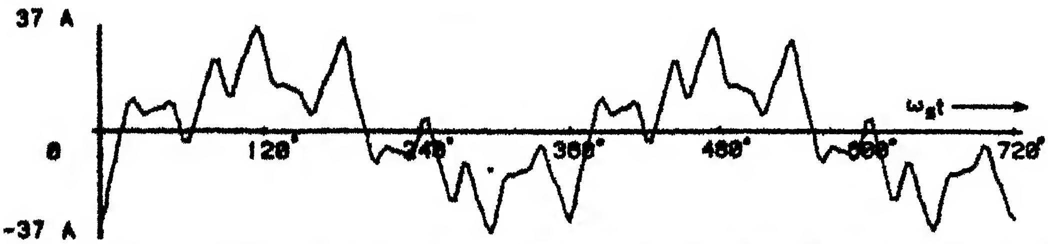
INSTANTANEOUS STATOR CURRENT AT SPEED 960 RPM FOR MOD INDEX = 1 0



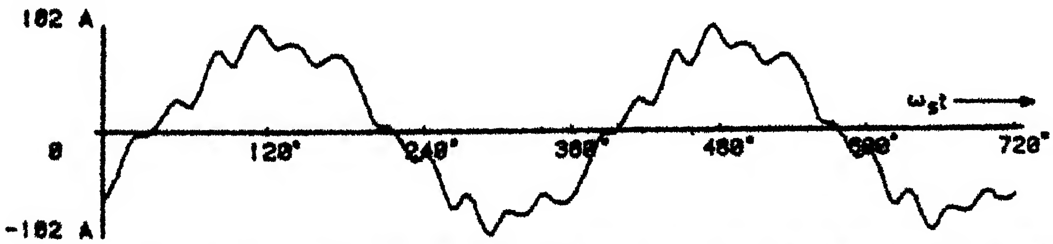
INSTANTANEOUS STATOR CURRENT AT SPEED 1176 RPM FOR MOD INDEX = 1 0



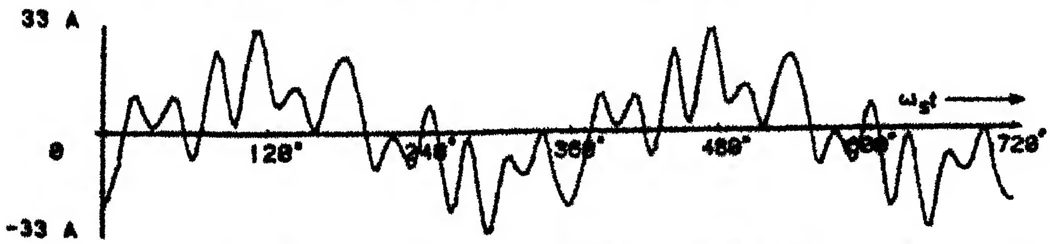
INSTANTANEOUS STATOR CURRENT AT SPEED 960 RPM FOR MOD INDEX = 0 8



INSTANTANEOUS STATOR CURRENT AT SPEED 1176 RPM FOR MOD INDEX = 0 8

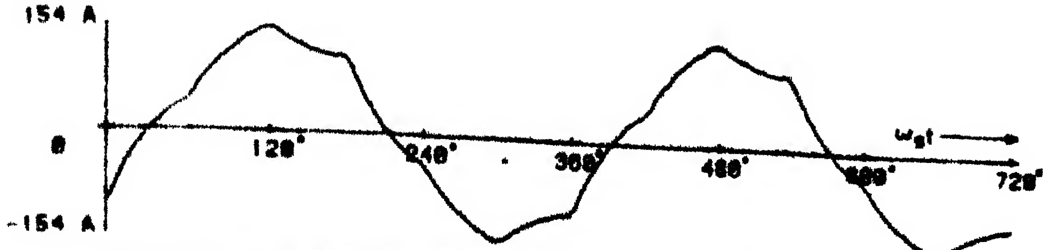


INSTANTANEOUS STATOR CURRENT AT SPEED 960 RPM FOR MOD INDEX = 0 6

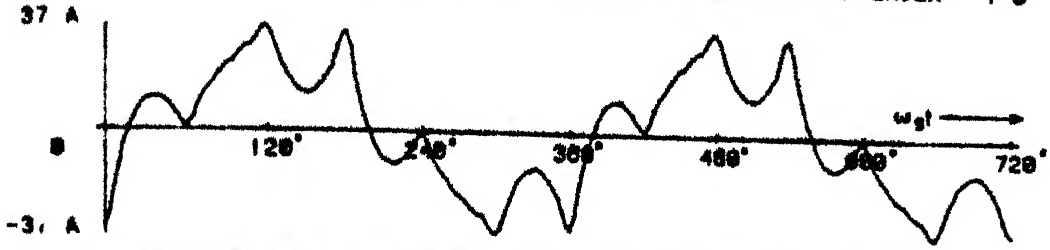


INSTANTANEOUS STATOR CURRENT AT SPEED 1176 RPM FOR MOD INDEX = 0 6

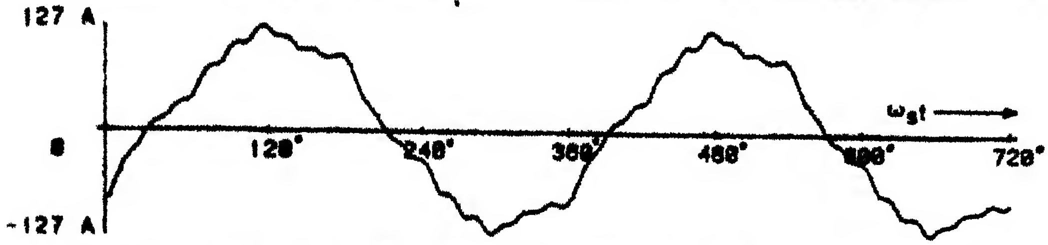
FIG 5 11(B) INST CURRENT WAVEFORMS FOR 6 PULSES/CYCLE AT INV FREQ = 60 HZ



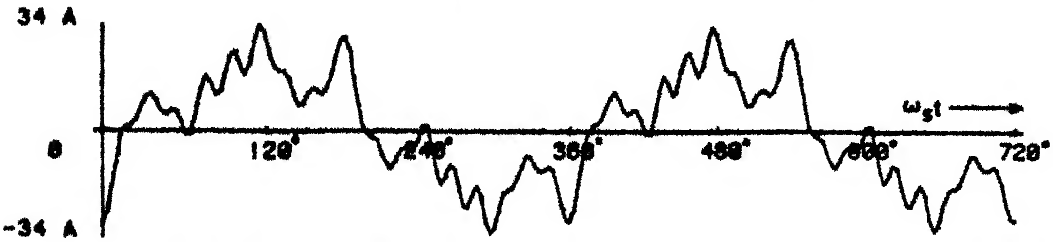
INSTANTANEOUS STATOR CURRENT AT SPEED 900 RPM FOR MOD INDEX = 1.0



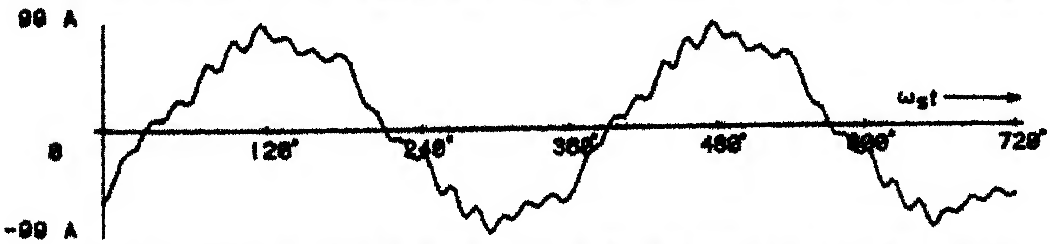
INSTANTANEOUS STATOR CURRENT AT SPEED 1176 RPM FOR MOD INDEX = 1.0



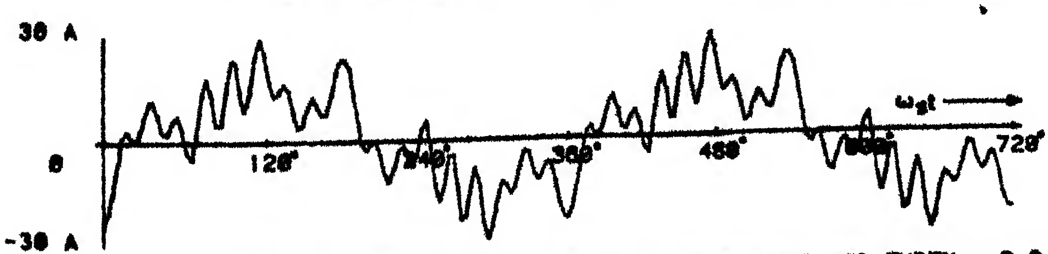
INSTANTANEOUS STATOR CURRENT AT SPEED 900 RPM FOR MOD INDEX = 0.8



INSTANTANEOUS STATOR CURRENT AT SPEED 1176 RPM FOR MOD INDEX = 0.8



INSTANTANEOUS STATOR CURRENT AT SPEED 900 RPM FOR MOD INDEX = 0.6



INSTANTANEOUS STATOR CURRENT AT SPEED 1176 RPM FOR MOD INDEX = 0.6

FIG 5.11(C) INST CURRENT WAVEFORMS FOR 9 PULSES/CYCLE AT INV FREQ = 60 HZ

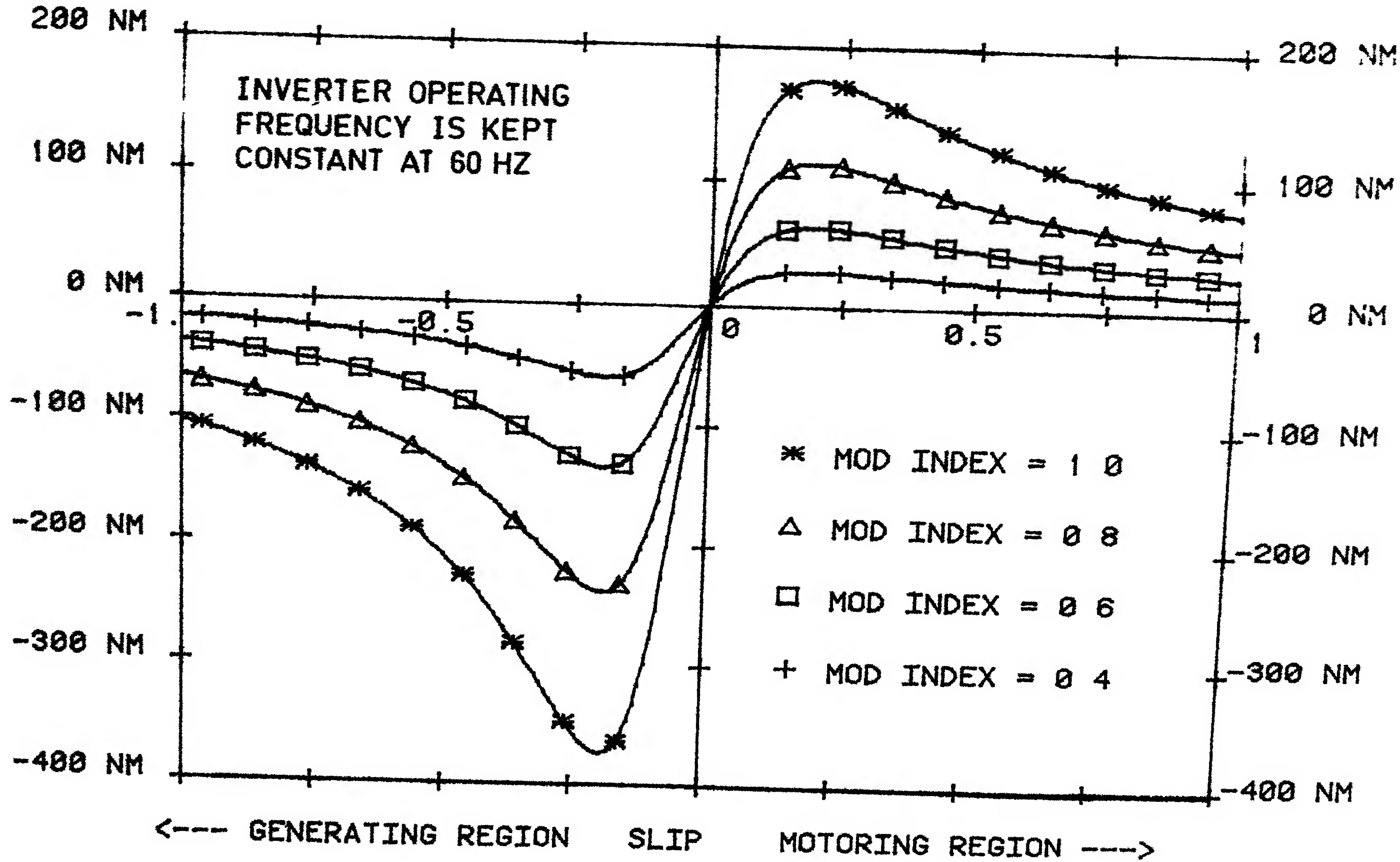


FIG 5 12(A) AV. TORQUE SLIP CHARACTERISTICS FOR VARING MOD.INDEX

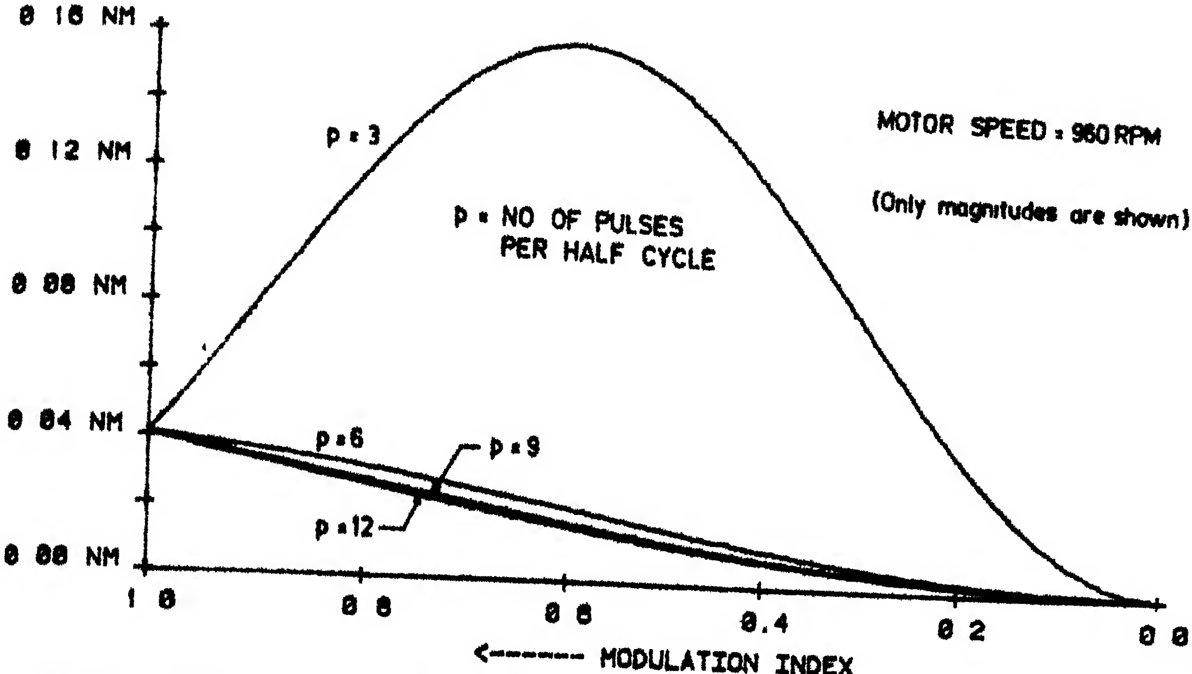


FIG 5 12(B) TYPICAL STEADY 5TH HAR TORQUE VS. MODINDEX CHARACTERISTICS

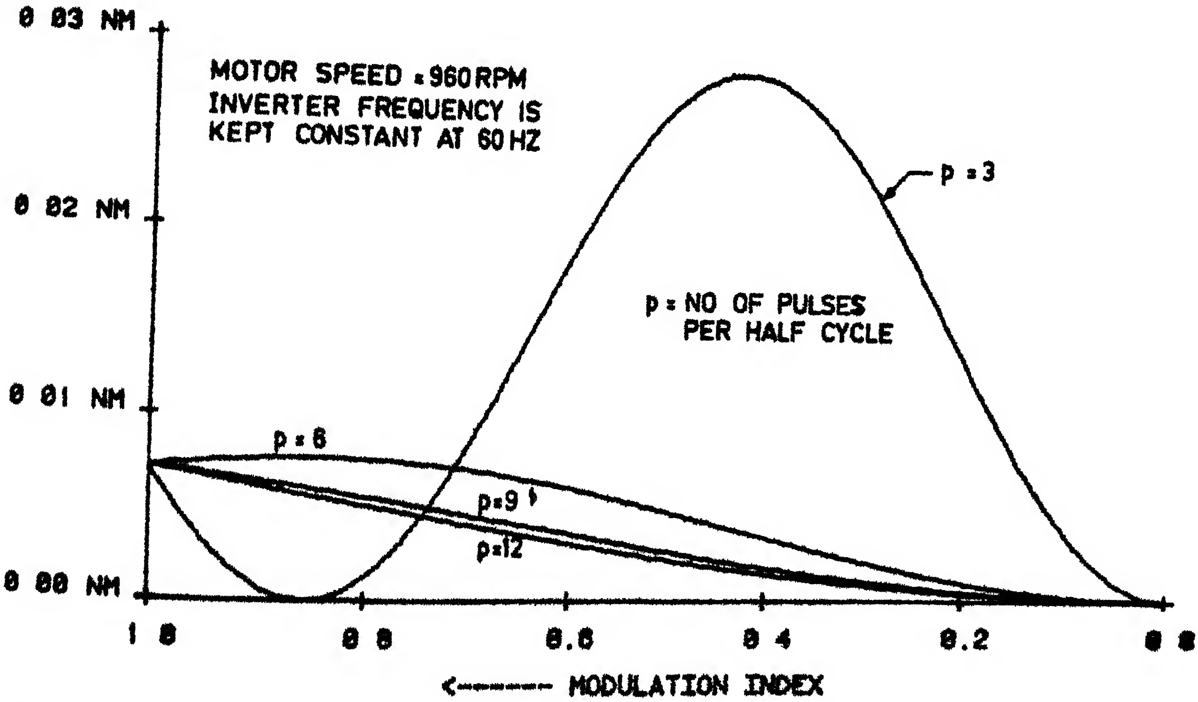


FIG 5 12(C) TYPICAL STEADY 7TH HAR.TORQUE VS MOD INDEX CHARACTERISTICS

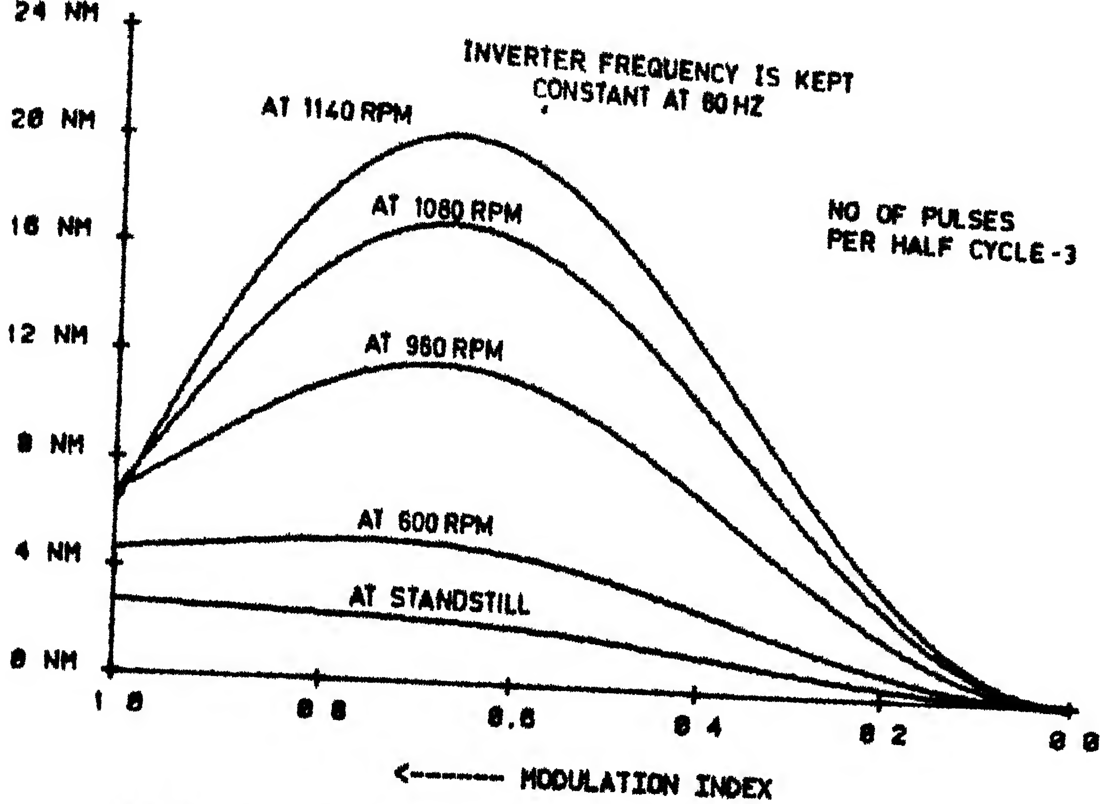


FIG 5.13(A) 6TH HARMONIC TORQUE VS. MOD INDEX CHARACTERISTICS

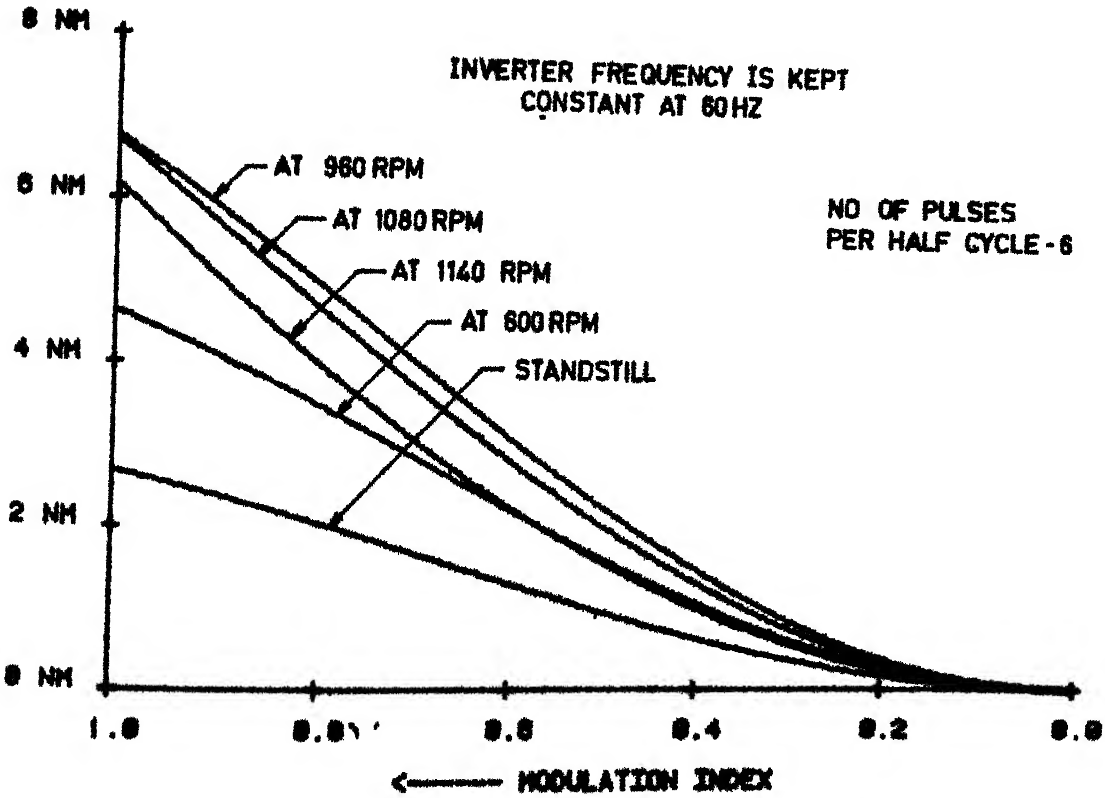


FIG 5.13(B) 6TH HARMONIC TORQUE VS. MOD.INDEX CHARACTERISTICS

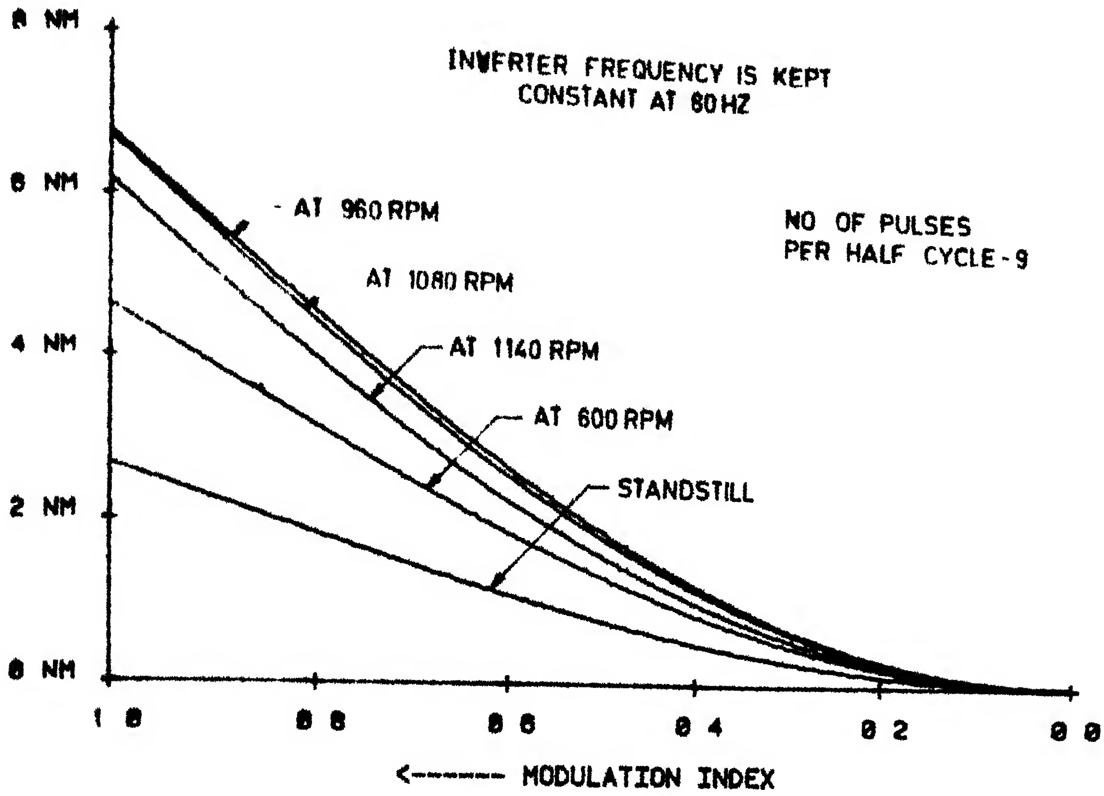


FIG 5 13(C) 6TH HARMONIC TORQUE VS. MOD INDEX CHARACTERISTICS

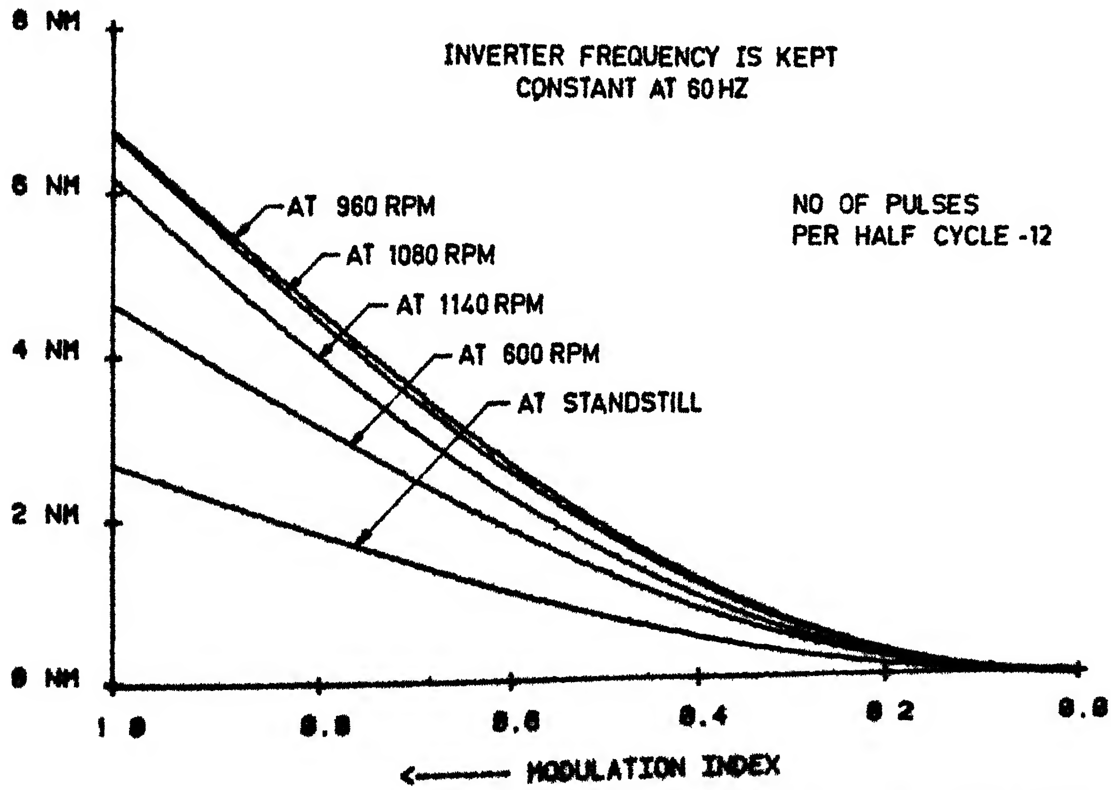


FIG 5 13(D) 6TH HARMONIC TORQUE VS. MOD.INDEX CHARACTERISTICS

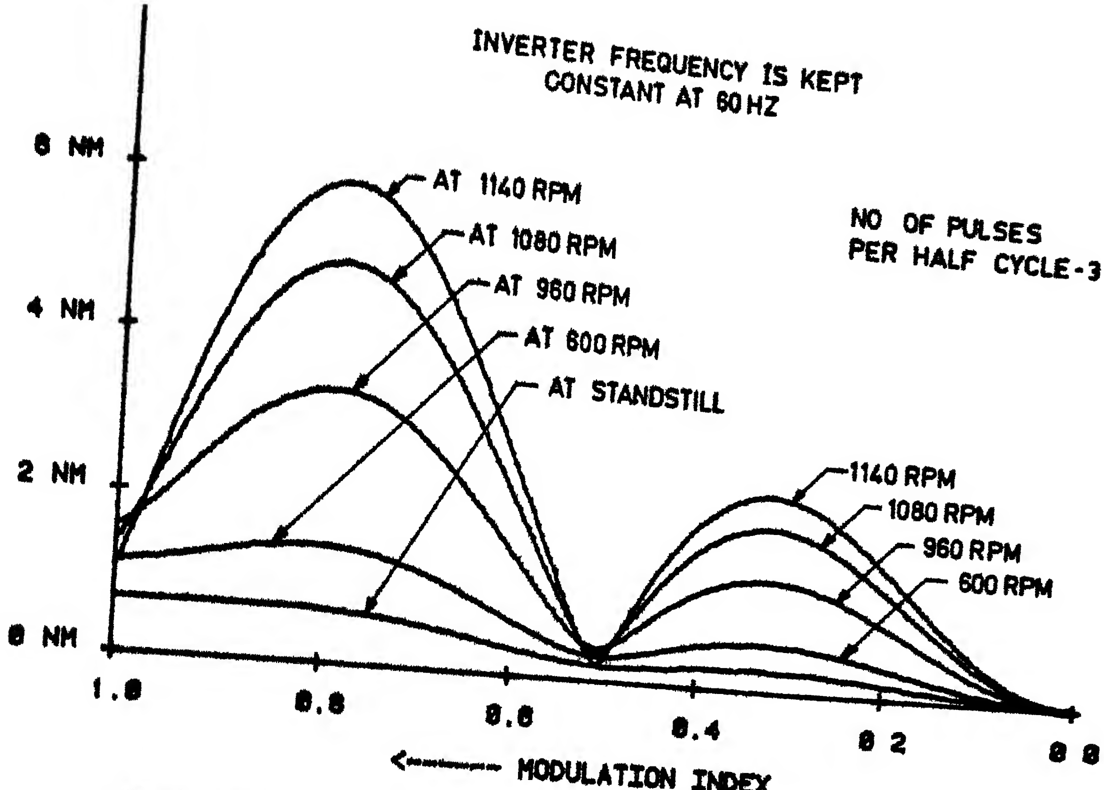


FIG 5.14(A) 12TH HARMONIC TORQUE VS. MOD INDEX CHARACTERISTICS

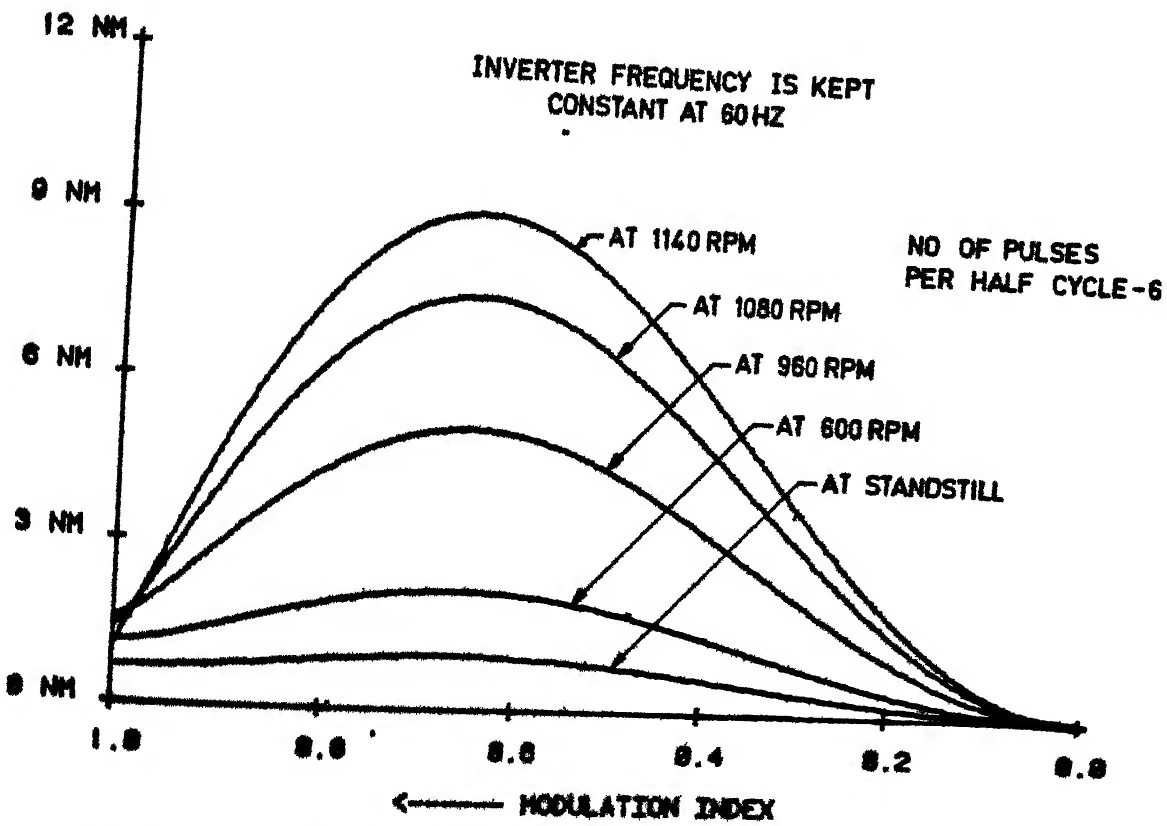


FIG 5.14(B) 12TH HARMONIC TORQUE VS. MOD. INDEX CHARACTERISTICS

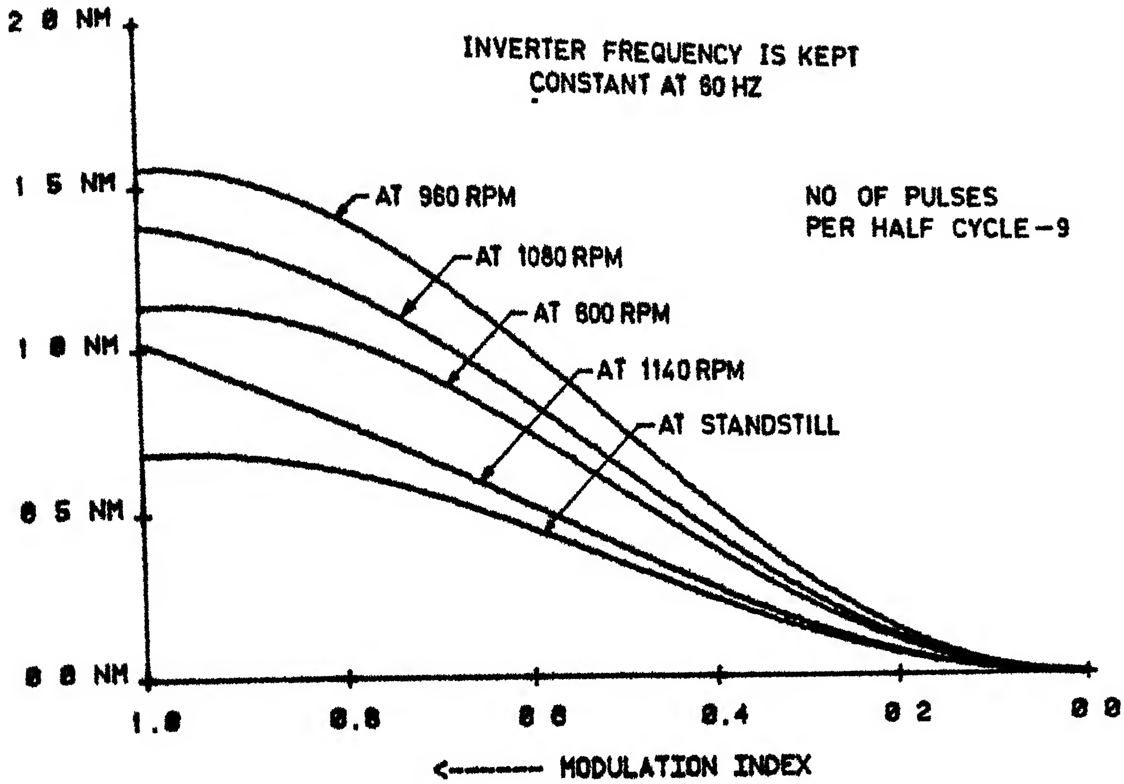


FIG 5 14(C) 12TH HARMONIC TORQUE VS. MOD INDEX CHARACTERISTICS

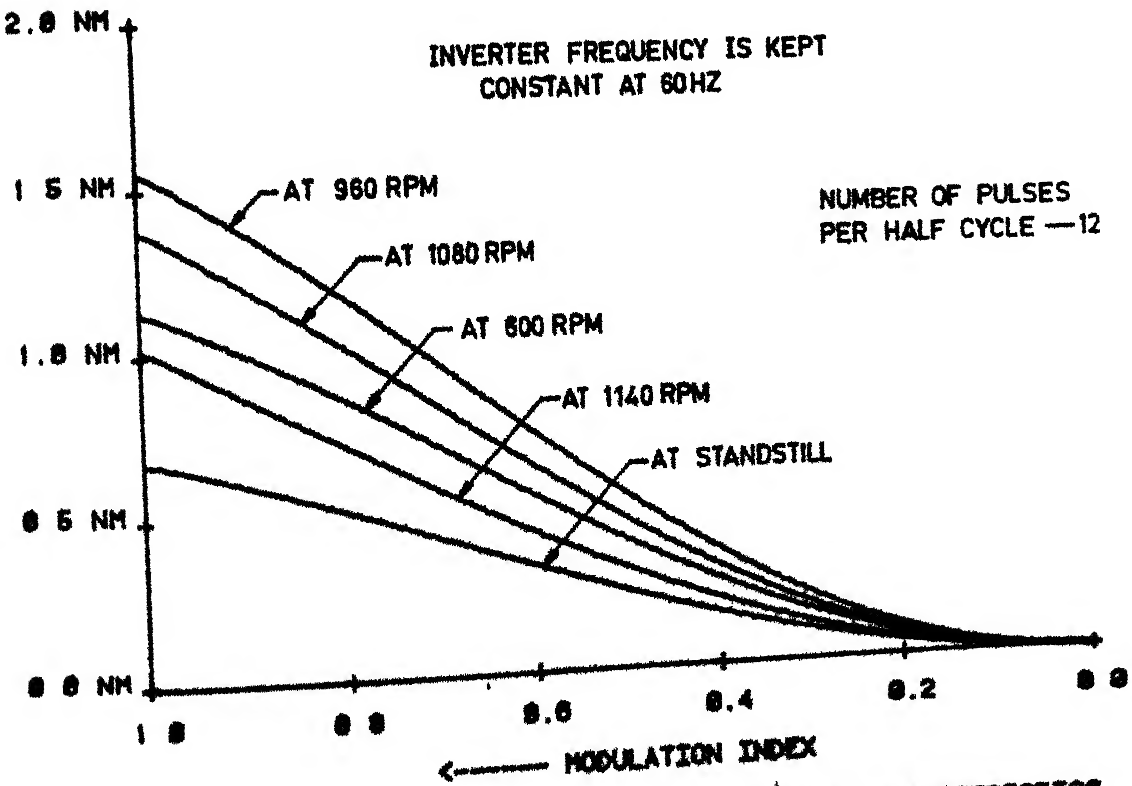
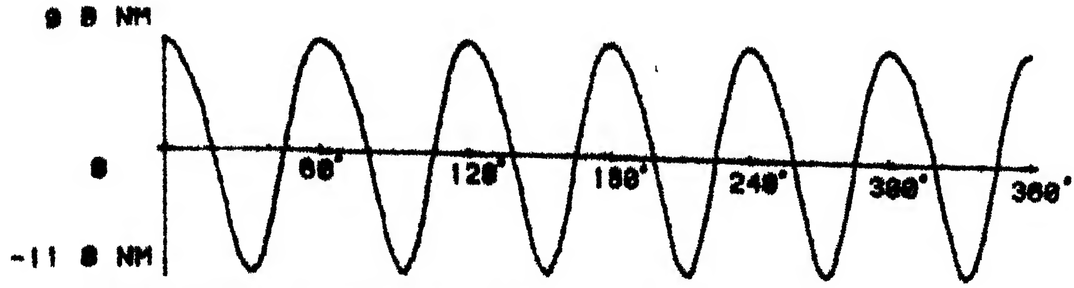
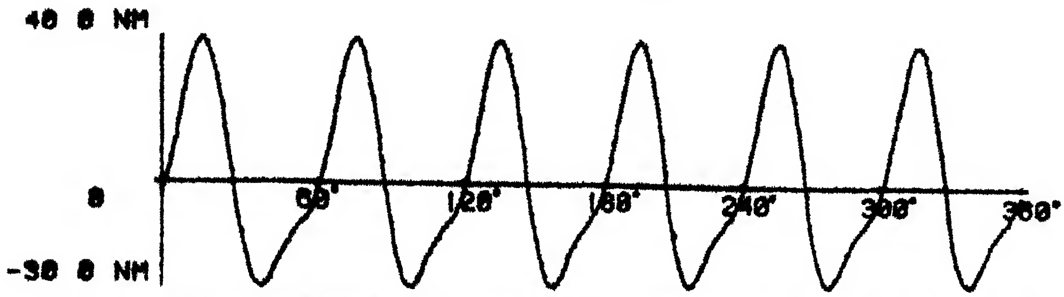


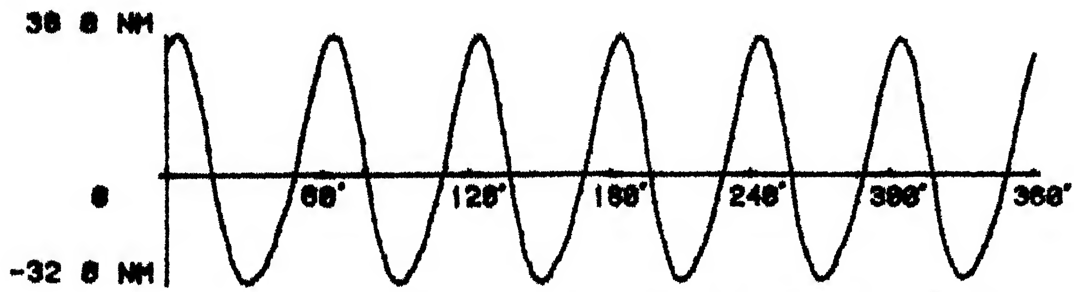
FIG 5 14(D) 12TH HARMONIC TORQUE VS. MOD.INDEX CHARACTERISTICS



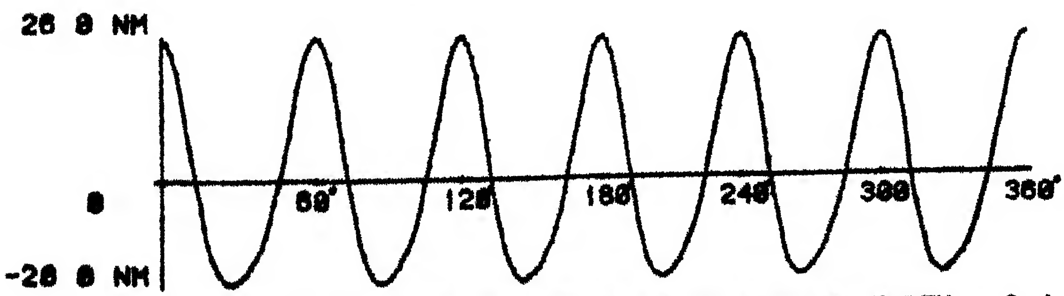
INSTANTANEOUS HARMONIC TORQUE AT 60 HZ FOR MOD INDEX = 1.0



INSTANTANEOUS HARMONIC TORQUE AT 60 HZ FOR MOD INDEX = 0.8



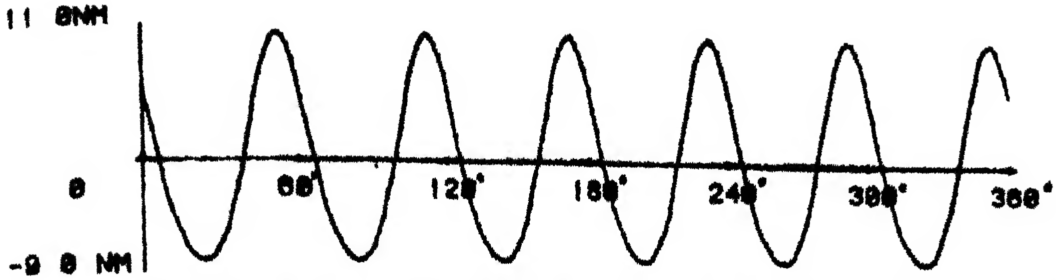
INSTANTANEOUS HARMONIC TORQUE AT 60 HZ FOR MOD INDEX = 0.6



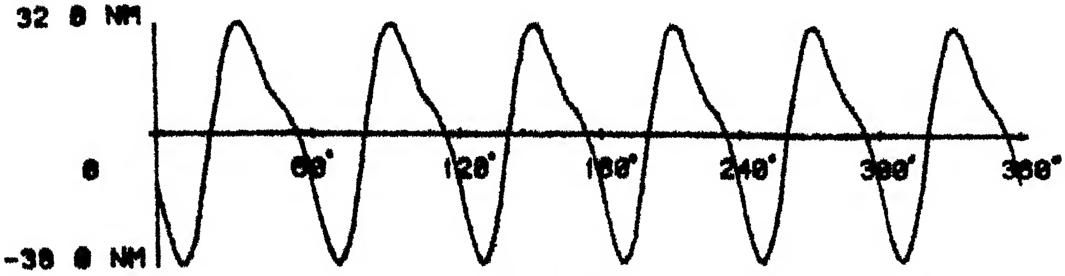
INSTANTANEOUS HARMONIC TORQUE AT 60 HZ FOR MOD INDEX = 0.4

$\omega_s t \longrightarrow$

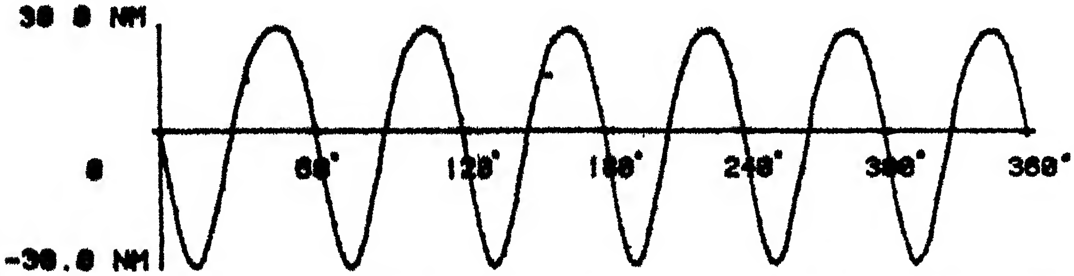
FIG 5.15(A) INST HAR TORQUE (1170 RPM) FOR 3 PULSES/CYCLE AT FREQ=60 HZ



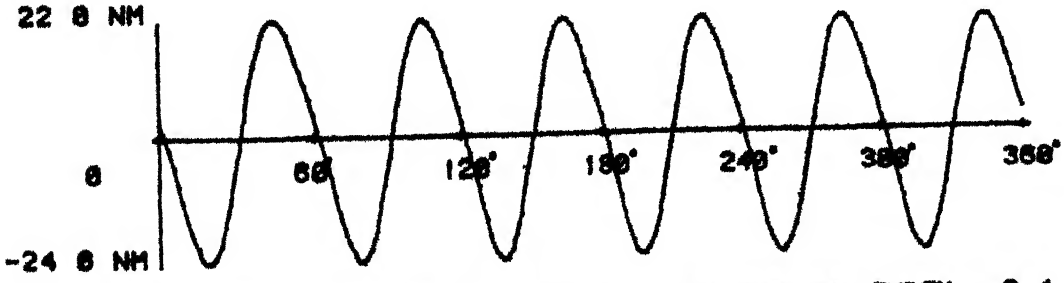
INSTANTANEOUS HARMONIC TORQUE AT 30 HZ FOR MOD INDEX = 1 0



INSTANTANEOUS HARMONIC TORQUE AT 30 HZ FOR MOD INDEX = 0 8



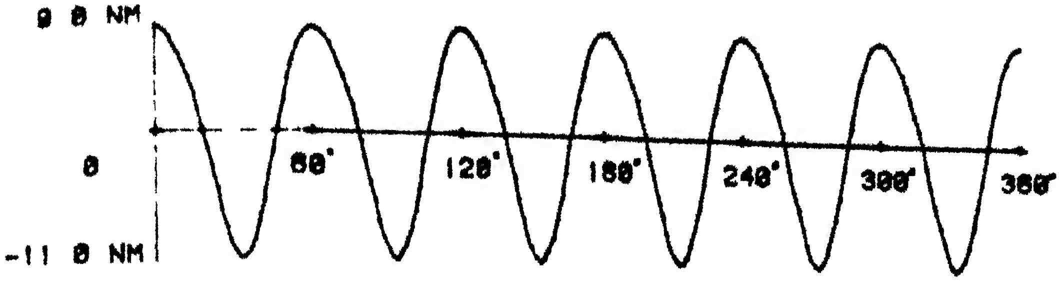
INSTANTANEOUS HARMONIC TORQUE AT 30 HZ FOR MOD INDEX = 0 6



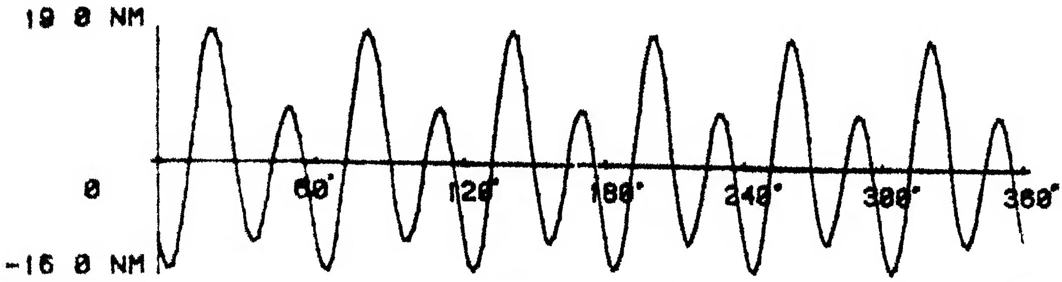
INSTANTANEOUS HARMONIC TORQUE AT 30 HZ FOR MOD INDEX = 0 4

$\omega_s t \longrightarrow$

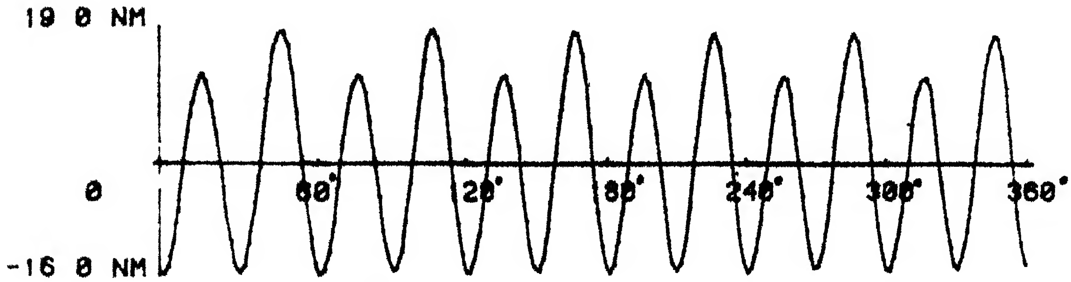
FIG 5 15(B) INST HAR TORQUE ([illegible] RPM) FOR 3 PULSES/CYCLE AT FREQ=30 HZ



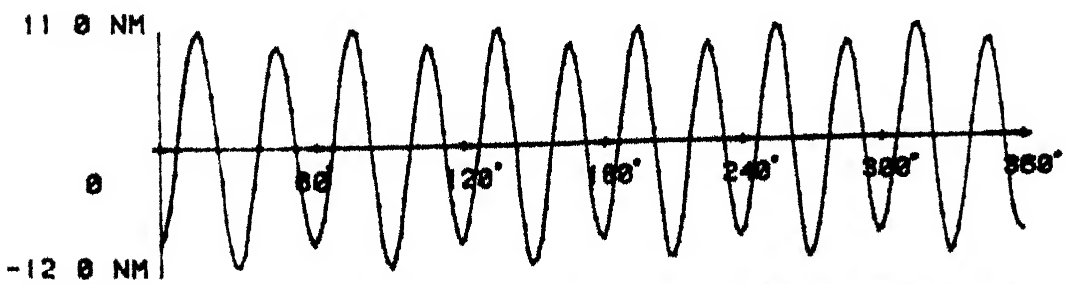
INSTANTANEOUS HARMONIC TORQUE AT 60 HZ FOR MOD INDEX = 1 0



INSTANTANEOUS HARMONIC TORQUE AT 60 HZ FOR MOD INDEX = 0 8



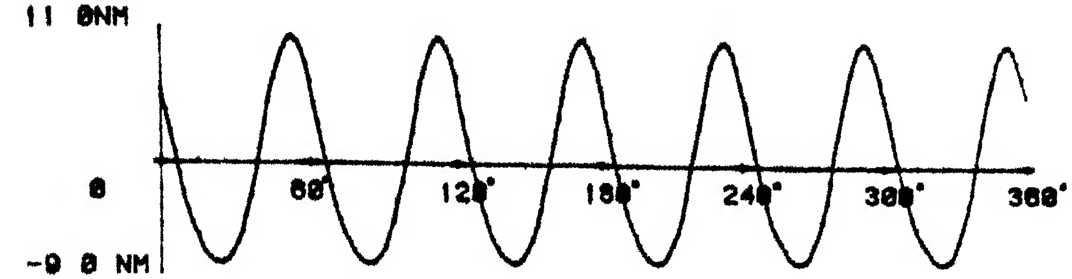
INSTANTANEOUS HARMONIC TORQUE AT 60 HZ FOR MOD INDEX = 0 6



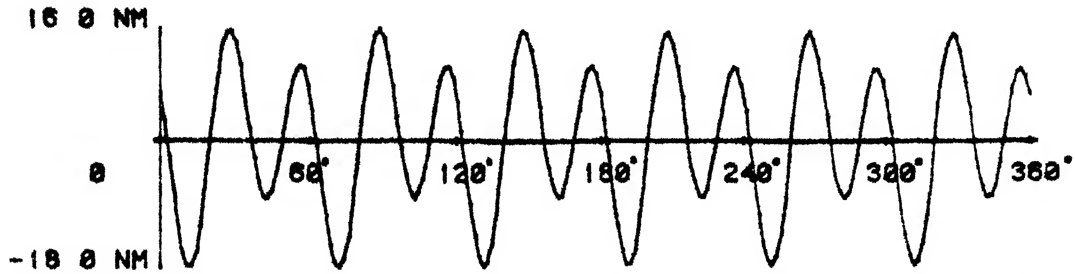
INSTANTANEOUS HARMONIC TORQUE AT 60 HZ FOR MOD INDEX = 0 4

$\omega_s t$ ⟶

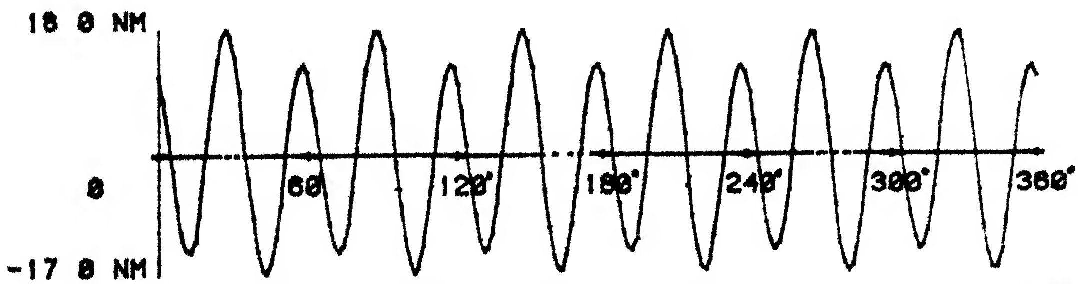
FIG 5 15(C) INST HAR TORQUE (1178 RPM) FOR 8 PULSES/CYCLE AT FREQ=60 HZ



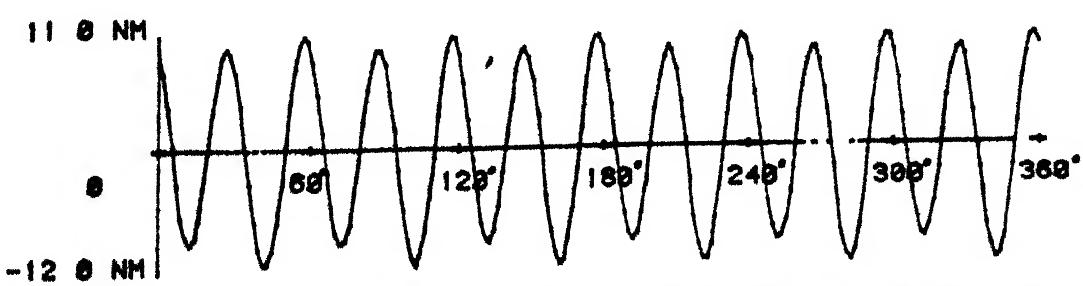
INSTANTANEOUS HARMONIC TORQUE AT 30 HZ FOR MOD INDEX = 1 0



INSTANTANEOUS HARMONIC TORQUE AT 30 HZ FOR MOD INDEX = 0 8



INSTANTANEOUS HARMONIC TORQUE AT 30 HZ FOR MOD INDEX = 0 6



INSTANTANEOUS HARMONIC TORQUE AT 30 HZ FOR MOD INDEX = 0 4

$\omega_s t \longrightarrow$

FIG 5 15(D) INST HAR TORQUE (500 RPM) FOR 6 PULSES/CYCLE AT FREQ=30 HZ

TABLE 5.1(a)

HARMONIC CONTENT IN VOLTAGE FOR E.P.W.M. VOLTAGE SOURCE INVERTER WITH VARYING MODULATING INDEX (NUMBER OF PULSES PER HALF CYCLE = 3)

Sl. No.	Modulation index (m_1)	Fundamental	Fifth Harmonic	Seventh Harmonic	Eleventh Harmonic	Thirteen Harmonic	Seven-teenth Harmonic	Nineteen Harmonic
1	1.00	1.00000	0.20000	0.14286	0.09091	0.07692	0.05882	0.05263
2	0.95	0.95432	0.24350	0.09537	0.13189	0.02804	0.09696	0.00276
3	0.90	0.90798	0.28284	0.04470	0.16200	0.02407	0.11620	0.04779
4	0.85	0.86102	0.31734	0.00748	0.47877	0.07341	0.11280	0.08675
5	0.80	0.81347	0.34641	0.05940	0.18082	0.11433	0.08743	0.10469
6	0.75	0.76537	0.36955	0.10934	0.16798	0.14214	0.04502	0.09725
7	0.70	0.71474	0.38637	0.15561	0.14130	0.15364	0.00616	0.06624
8	0.65	0.66761	0.39658	0.19667	0.10298	0.14751	0.05614	0.01918
9	0.60	0.61803	0.40000	0.23115	0.05618	0.12446	0.09518	0.03253
10	0.55	0.56803	0.39658	0.25788	0.00476	0.08714	0.11568	0.07636
11	0.50	0.51764	0.38637	0.27598	0.04706	0.03982	0.11364	0.10168
12	0.45	0.46639	0.36955	0.28483	0.09500	0.01207	0.08946	0.10235
13	0.40	0.41582	0.34641	0.28415	0.13512	0.05257	0.04785	0.07823
14	0.35	0.36447	0.31734	0.27395	0.16411	0.10590	0.00308	0.03514
15	0.30	0.31287	0.28284	0.25457	0.17958	0.13708	0.05341	0.01647
16	0.25	0.26105	0.24350	0.22667	0.18026	0.16258	0.09334	0.06408
17	0.20	0.20906	0.20000	0.19118	0.16610	0.15048	0.11508	0.09616
18	0.15	0.15692	0.15307	0.14929	0.13826	0.13118	0.11440	0.10494
19	0.10	0.10467	0.10353	0.10239	0.09903	0.09582	0.09143	0.08828
20	0.05	0.05285	0.05221	0.05207	0.05164	0.05185	0.05065	0.05023
21	0.00	0.0000	0.0000	0.00000	0.00000	0.00000	0.00000	0.00000

(Note: All values are in p.u.).

TABLE 5.1 (b)

HARMONIC CONTENT IN VOLTAGE FOR E.P.W.M. VOLTAGE SOURCE INVERTER WITH VARYING MODULATING INDEX
(NUMBER OF PULSES PER HALF CYCLE = 6)

Sl. No.	Modulation index (m_1)	Fundamental	Fifth Harmonic	Seventh Harmonic	Eleventh Harmonic	Thirteen Harmonic	Seventeenth Harmonic	Nineteenth Harmonic
1	1.00	1.00000	0.20000	0.14286	0.09091	0.07692	0.05882	0.05263
2	0.95	0.95106	0.19607	0.14576	0.13865	0.02720	0.05389	0.05448
3	0.90	0.90196	0.19129	0.14744	0.18353	0.02332	0.04631	0.05298
4	0.85	0.85271	0.18570	0.14788	0.22463	0.07316	0.03644	0.04822
5	0.80	0.80381	0.17932	0.14709	0.26103	0.12089	0.02477	0.04049
6	0.75	0.75377	0.17216	0.14505	0.29205	0.16512	0.01180	0.03027
7	0.70	0.70410	0.16427	0.14181	0.31703	0.20458	0.00159	0.01819
8	0.65	0.65432	0.15567	0.13737	0.33545	0.23814	0.01499	0.00499
9	0.60	0.60442	0.14641	0.13178	0.34692	0.25481	0.02765	0.00852
10	0.55	0.55441	0.13652	0.12508	0.35122	0.28384	0.03894	0.02151
11	0.50	0.50431	0.12605	0.11733	0.34824	0.29457	0.04831	0.03317
12	0.45	0.45413	0.11503	0.10860	0.33806	0.29698	0.05530	0.04279
13	0.40	0.40387	0.10353	0.09896	0.32088	0.29071	0.05957	0.04978
14	0.35	0.35364	0.09158	0.08849	0.29706	0.27605	0.06089	0.05370
15	0.30	0.30314	0.07924	0.07728	0.26709	0.25341	0.05922	0.05432
16	0.25	0.25270	0.06656	0.06541	0.23159	0.22345	0.05462	0.05160
17	0.20	0.20221	0.05359	0.05300	0.19130	0.18704	0.04733	0.04570
18	0.15	0.15169	0.04039	0.04015	0.14705	0.14522	0.03770	0.03699
19	0.10	0.10114	0.02703	0.02695	0.09976	0.09921	0.02622	0.02600
20	0.05	0.05057	0.01354	0.01353	0.05040	0.05033	0.01344	0.01341
21	0.00	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

(Note All values are in p.u.).

TABLE 5.1 (c)

HARMONIC CONTENT IN VOLTAGE FOR E.P.W.M. VOLTAGE SOURCE INVERTER WITH VARYING MODULATION INDEX
(NUMBER OF PULSES PER HALF CYCLE = 5)

Sl. No.	Modulation index (m_1)	Fundamental	Fifth Harmonic	Seventh Harmonic	Eleventh Harmonic	Thirteen Harmonic	Seventeenth Harmonic	Nineteenth Harmonic
1	1.00	1.00000	0.20000	0.14286	0.09091	0.07692	0.05882	0.05263
2	0.95	0.95047	0.19249	0.13942	0.09366	0.08374	0.10749	0.00265
3	0.90	0.90087	0.18461	0.13546	0.09555	0.08947	0.15379	0.04741
4	0.85	0.85120	0.17638	0.13099	0.09656	0.09406	0.19671	0.09617
5	0.80	0.80147	0.16782	0.12603	0.09668	0.09743	0.23532	0.14229
6	0.75	0.75167	0.15894	0.12061	0.09592	0.09956	0.26875	0.18451
7	0.70	0.70182	0.14975	0.11473	0.09426	0.10040	0.29628	0.22167
8	0.65	0.65191	0.14028	0.10843	0.09174	0.09995	0.31730	0.25275
9	0.60	0.60196	0.13054	0.10172	0.08838	0.09822	0.33135	0.27689
10	0.55	0.55195	0.12055	0.09461	0.08420	0.09528	0.33812	0.29344
11	0.50	0.50191	0.11034	0.08720	0.07925	0.09101	0.33746	0.30194
12	0.45	0.45183	0.09991	0.07943	0.07356	0.08562	0.32939	0.30216
13	0.40	0.40171	0.08930	0.07137	0.06720	0.07913	0.31408	0.29409
14	0.35	0.35156	0.07851	0.06304	0.06022	0.07162	0.29188	0.27795
15	0.30	0.30139	0.06757	0.05448	0.05269	0.06319	0.26326	0.25420
16	0.25	0.25119	0.05651	0.04571	0.04467	0.05395	0.22886	0.22346
17	0.20	0.20098	0.04534	0.03678	0.03624	0.04402	0.18943	0.18660
18	0.05	0.15075	0.03408	0.02770	0.02743	0.03362	0.14584	0.14462
19	0.10	0.10050	0.02275	0.01853	0.01846	0.02259	0.09904	0.09868
20	0.50	0.05025	0.01139	0.00928	0.00927	0.01137	0.05007	0.05002
21	0.00	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

(Note: All values are in p.u.).

TABLE 5.1 (d)

HARMONIC CONTENT IN VOLTAGE FOR E.P.W.M. VOLTAGE SOURCE INVERTER WITH VARYING MODULATION INDEX
(NUMBER OF PULSES PER HALF CYCLE = 1)

Sl. No.	Modulation index (m_1)	Fundamental	Fifth Harmonic	Seventh Harmonic	Eleventh Harmonic	Thirteen Harmonic	Seventeenth Harmonic	Nineteenth Harmonic
1	1.00	1.00000	0.200000	0.14286	0.09091	0.07692	0.05882	0.05263
2	0.95	0.95207	0.19136	0.13769	0.08981	0.07751	0.06347	0.06073
3	0.90	0.90049	0.18252	0.13223	0.08825	0.07758	0.06733	0.06790
4	0.85	0.85067	0.17349	0.12649	0.08623	0.07699	0.07037	0.07401
5	0.80	0.80082	0.16427	0.12049	0.08377	0.07589	0.07253	0.07898
6	0.75	0.75094	0.15487	0.11423	0.08087	0.07425	0.07379	0.08273
7	0.70	0.70102	0.14531	0.10774	0.07755	0.07206	0.07414	0.08521
8	0.65	0.65107	0.13559	0.10102	0.07383	0.06936	0.07357	0.08637
9	0.60	0.60110	0.12573	0.09409	0.06972	0.06615	0.07210	0.08619
10	0.55	0.55110	0.11573	0.08695	0.06526	0.05247	0.06973	0.08468
11	0.50	0.50107	0.10560	0.07964	0.06046	0.05888	0.06650	0.08187
12	0.45	0.45103	0.09537	0.07216	0.05534	0.05377	0.06245	0.07779
13	0.40	0.40096	0.08503	0.06453	0.04994	0.04883	0.05762	0.07251
14	0.35	0.35088	0.07460	0.05676	0.04428	0.04368	0.05208	0.06611
15	0.30	0.30078	0.06409	0.04888	0.03839	0.03791	0.04590	0.05869
16	0.25	0.25067	0.05352	0.04089	0.03230	0.03202	0.03915	0.05036
17	0.20	0.20055	0.04288	0.03281	0.02604	0.02590	0.03192	0.04125
18	0.15	0.15042	0.03220	0.02467	0.01965	0.01959	0.02429	0.03151
19	0.10	0.10028	0.02149	0.01648	0.01316	0.01314	0.01636	0.02128
20	0.05	0.05014	0.01075	0.00825	0.00660	0.00659	0.00823	0.01072

(Note: All values are in p.u.).

TABLE 5.2

HARMONIC CONTENT IN PHASE VOLTAGES FOR E.P.W.M. VOLTAGE SOURCE INVERTER FOR VARYING MODULATING INDEX (NUMBER OF PULSES PER HALF CYCLE = 4)

Phase	Modulation index (m_1)	Fundamental	Third Harmonic	Fifth Harmonic	Seventh Harmonic	Ninetn Harmonic
Phase 'a'	1.000	1.0000	0.0000	0.2000	0.1428	0.0000
	0.900	0.9101	0.0137	0.1822	0.2275	0.0921
	0.800	0.8193	0.0242	0.1599	0.2987	0.1820
	0.700	0.7278	0.0313	0.1341	0.3510	0.2585
	0.600	0.6357	0.0348	0.1057	0.3805	0.3122
	0.500	0.5431	0.0348	0.0758	0.3851	0.3365
	0.400	0.4502	0.0313	0.0455	0.3643	0.3284
	0.300	0.3495	0.0243	0.0234	0.3119	0.2883
	0.200	0.2333	0.0164	0.0161	0.2219	0.2146
	0.100	0.1167	0.0082	0.0082	0.1153	0.1143
Phase 'b'	1.000	1.0000	0.0000	0.2000	0.1428	0.0000
	0.900	0.8954	0.0176	0.1815	0.1176	0.1036
	0.800	0.7909	0.0344	0.1625	0.1596	0.1965
	0.700	0.6866	0.0502	0.1437	0.2181	0.2673
	0.600	0.5826	0.0650	0.1262	0.2644	0.3075
	0.500	0.4793	0.0786	0.1110	0.2882	0.3123
	0.400	0.3770	0.0910	0.0993	0.2857	0.2815
	0.300	0.2787	0.0891	0.0858	0.2488	0.2299
	0.200	0.1860	0.0601	0.0591	0.1770	0.1711
	0.100	0.0931	0.0302	0.0301	0.0919	0.0912

(Note: All values are in p.u.)

Continued.....

TABLE 5.2 (CONTINUED):

Phase	Modulation index (m_i)	Fundamental	Third Harmonic	Fifth Harmonic	Seventh Harmonic	Nineth Harmonic
Phase 'c'	1.000	1.0000	0.0000	0.2000	0.1428	0.0000
	0.900	0.8954	0.0176	0.1815	0.1176	0.1036
	0.800	0.7909	0.0344	0.1625	0.1596	0.1965
	0.700	0.6866	0.0502	0.1437	0.2181	0.2673
	0.600	0.5826	0.0650	0.1262	0.2644	0.3075
	0.500	0.4793	0.0786	0.1110	0.2882	0.3123
	0.400	0.3770	0.0910	0.0993	0.2857	0.2815
	0.300	0.2787	0.0891	0.0858	0.2488	0.2299
	0.200	0.1860	0.0601	0.0591	0.1770	0.1711
	0.100	0.0931	0.0302	0.0301	0.0919	0.0912

(Note: All values are in p.u.)

TABLE 5.3 (a)

VARIATION OF CURRENT HARMONIC FACTOR WITH MODULATION INDEX FOR DIFFERENT NUMBER OF PULSES AND MOTOR SPEED (INVERTER OPERATING FREQUENCY = 60 Hz)

	Modulation index (m_i)	3 pulses per half cycle	6 pulses per half cycle	9 pulses per half cycle	12 pulses per half cycle
Fundamental slip=0.5	1.00	0.0594215	0.0594215	0.0594215	0.0594215
	0.90	0.0839204	0.0673189	0.0629396	0.0614005
	0.80	0.1145554	0.0789929	0.0686182	0.0640925
	0.70	0.1475165	0.0933522	0.0764163	0.0674935
	0.60	0.1815932	0.1093407	0.0857713	0.0714404
	0.50	0.2165131	0.1262742	0.0958179	0.0756463
	0.40	0.2519411	0.1434846	0.1056087	0.07977661
	0.30	0.2877185	0.1598391	0.1142543	0.0834061
	0.20	0.3243839	0.1736248	0.1210031	0.0862706
	0.10	0.3564031	0.1829190	0.1252877	0.0880963
	0.00	0.2822757	0.1953510	0.3015775	0.0990430
Fundamental slip=0.2	1.00	0.0803719	0.0803719	0.0803719	0.0803719
	0.90	0.1135031	0.0910565	0.0851316	0.0830488
	0.80	0.1549369	0.1068522	0.0928155	0.0866918
	0.70	0.1995181	0.1262815	0.1033683	0.0912941
	0.60	0.2456096	0.1479151	0.1160273	0.0966354
	0.50	0.2928437	0.1708274	0.1296220	0.1023269
	0.40	0.3407681	0.1941140	0.1423703	0.1078885
	0.30	0.3891691	0.2162426	0.1545692	0.1128276
	0.20	0.4387759	0.2348954	0.1637010	0.1167039
	0.10	0.4820998	0.2474708	0.1694985	0.1191743
	0.00	0.3818597	0.2642801	0.4079411	0.1339641

Continued.....

TABLE 5.3 (a) (Continued):

	Modulation index (m_1)	3 pulses per half cycle	6 pulses per half cycle	9 pulses per half cycle	12 pulses per half cycle
Fundamental slip = 0.02	1.00	0.4385093	0.4385094	0.4385094	0.4385094
	0.90	0.6192559	0.4968173	0.4644350	0.4531193
	0.80	0.8453115	0.5830208	0.6064215	0.4730012
	0.70	1.0885440	0.6890552	0.5640165	0.4981191
	0.60	1.3400211	0.8071191	0.6331060	0.5272716
	0.50	1.5977409	0.9321600	0.7073012	0.5583357
	0.40	1.8592392	1.0592444	0.7796059	0.5886895
	0.30	2.1233513	1.1800083	0.8434530	0.6156460
	0.20	2.3940621	1.2818035	0.8932904	0.6368019
	0.10	2.6304970	1.3504320	0.9249305	0.6502854
	0.00	2.0836683	1.4421216	2.2258714	0.7309167

TABLE 5.3 (b)

VARIATION OF CURRENT HARMONIC FACTOR WITH MODULATION INDEX FOR DIFFERENT NUMBER OF PULSES AND MOTOR SPEED (INVERTER OUTPUT VOLTAGE TO FREQUENCY RATIO IS CONSTANT)

Pulse number	Modulation index (m_1)	3 pulses per half cycle	6 pulses per half cycle	9 pulses per half cycle	12 pulses per half cycle
Fundamental slip=0.5	1.00	0.0594215	0.0594215	0.0594215	0.0594215
	0.90	0.0876283	0.0703128	0.0657349	0.0641253
	0.80	0.1262965	0.0871905	0.0757199	0.0707080
	0.70	0.1742952	0.1106220	0.0905122	0.0798753
	0.60	0.2346480	0.1421227	0.1114281	0.0926304
	0.50	0.3147135	0.1855253	0.1407139	0.1106783
	0.40	0.4286266	0.2488789	0.1831279	0.1373726
	0.30	0.6085023	0.3497301	0.2499968	0.1802395
	0.20	0.9356924	0.5349142	0.3733837	0.2594489
	0.10	1.6347103	0.9783219	0.6830119	0.4517558
	0.00	2.2682590	1.7836902	2.6624445	0.8141651
Fundamental slip=0.2	1.00	0.0803719	0.0803719	0.0803719	0.0803719
	0.90	0.1218850	0.0978093	0.0914396	0.0891997
	0.80	0.1810204	0.1249947	0.1085458	0.1013571
	0.70	0.2578205	0.1636895	0.1339238	0.1181746
	0.60	0.3584563	0.2172270	0.1702989	0.1415468
	0.50	0.4961428	0.2927148	0.2219937	0.1745621
	0.40	0.6955927	0.4041960	0.2973820	0.2229860
	0.30	1.0086392	0.5808250	0.4151354	0.2990927
	0.20	1.5594631	0.8943636	0.6242030	0.4332088
	0.10	2.5676680	1.5445541	1.0787711	0.7122140
	0.00	2.2682590	1.7836902	2.6624445	0.8141651

Continued......2

TABLE 5.3 (b) CONTINUED:

	Modulation index (m_1)	3 pulses per half cycle	6 pulses per half cycle	9 pulses per half cycle	12 pulses per half cycle
Fundamental slip = 0.02	1.00	0.4385093	0.4385094	0.4385094	0.4385094
	0.90	0.6690095	0.5368967	0.5019251	0.4896268
	0.80	0.9902486	0.6838612	0.5938483	0.5545044
	0.70	1.3870078	0.8808142	0.7206114	0.6358295
	0.60	1.8602146	1.1277160	0.8840454	0.7347062
	0.50	2.4134705	1.4246879	1.0804122	0.8494148
	0.40	3.0347382	1.7649080	1.2984140	0.9733108
	0.30	3.6752003	2.1191546	1.5144891	1.0906318
	0.20	4.1939223	2.4104567	1.6821577	1.1664964
	0.10	4.0155970	2.4237795	1.6932814	1.1165666
	0.00	2.2682590	1.7836902	2.6624445	0.8141651

TABLE 5.4 (a)

VARIATION OF SIXTH HARMONIC TORQUE WITH MODULATION INDEX FOR DIFFERENT MOTOR SPEEDS
(INVERTER OUTPUT VOLTAGE TO FREQUENCY RATIO IS CONSTANT)

	Mod. index	At standstill	At 600 rpm	At 960 rpm	At 1080 rpm	At 1140 rpm	At 1176 rpm
p = 3	1.0	2.689	4.638	6.774	6.725	6.192	6.070
	0.9	3.334	6.330	11.326	14.350	16.204	17.415
	0.8	4.609	9.429	18.595	24.713	28.408	30.624
	0.7	6.898	14.197	27.913	36.741	41.948	45.036
	0.6	10.383	20.594	38.622	49.601	55.987	59.813
	0.5	15.054	28.297	49.849	62.292	69.505	73.913
	0.4	20.631	36.560	60.373	73.536	81.208	86.012
	0.3	26.294	43.955	68.372	81.465	89.186	94.143
	0.2	30.042	47.586	70.329	82.287	89.425	94.099
	0.1	26.246	39.496	55.652	63.889	68.781	71.991
	0.0	0.000	0.000	0.0000	0.000	0.000	0.000
p = 6	1.0	2.689	4.638	6.774	6.725	6.192	6.070
	0.9	3.069	5.118	6.999	6.645	5.999	5.882
	0.8	3.431	5.505	7.021	6.403	5.720	5.662
	0.7	3.739	5.737	6.800	5.992	5.368	5.424
	0.6	3.938	5.738	6.298	5.414	4.960	5.186
	0.5	3.948	5.413	5.487	4.689	4.535	4.978
	0.4	3.660	4.659	4.354	3.874	4.160	4.841
	0.3	2.942	3.375	2.938	3.150	3.971	4.845
	0.2	1.707	1.490	1.722	3.032	4.191	5.103
	0.1	0.637	0.775	2.523	3.851	4.796	5.489
	0.0	0.000	0.000	0.0000	0.000	0.000	0.000

(All quantities are in Newton-meter)

Continued....

TABLE 5.4 (a) Continued):

	Mod. index	At standstill	At 600 rpm	At 960 rpm	At 1080 rpm	At 1140 rpm	At 1176 rpm
p = 9	1.0	2.689	4.638	6.774	6.725	6.192	6.070
	0.9	2.900	4.860	6.726	6.520	6.039	6.007
	0.8	3.089	5.006	6.549	6.240	5.861	5.936
	0.7	3.232	5.037	6.228	5.889	5.663	5.861
	0.6	3.292	4.908	5.754	5.479	5.454	5.781
	0.5	3.213	4.560	5.124	5.028	5.243	5.698
	0.4	2.919	3.933	4.353	4.561	5.036	5.607
	0.3	2.318	2.977	3.493	4.120	4.835	5.490
	0.2	1.325	1.706	2.708	3.755	4.612	5.283
	0.1	0.031	0.843	2.371	3.426	4.165	4.703
	0.0	0.000	0.000	0.000	0.000	0.000	0.000
p = 12	1.0	2.689	4.638	6.774	6.725	6.192	6.070
	0.9	2.852	4.785	6.643	6.474	6.033	6.020
	0.8	2.994	4.864	6.408	6.173	5.864	5.967
	0.7	3.094	4.8436	6.059	5.829	5.687	5.910
	0.6	3.119	4.682	5.591	5.450	5.508	5.850
	0.5	3.021	4.333	5.007	5.051	5.328	5.781
	0.4	2.733	3.747	4.321	4.648	5.146	5.693
	0.3	2.173	2.885	3.574	4.254	4.945	5.556
	0.2	1.269	1.773	2.865	3.864	4.659	5.277
	0.1	0.162	0.936	2.363	3.340	4.022	4.518
	0.0	0.000	0.000	0.000	0.000	0.000	0.000

(All quantities are in Newton-meter)

TABLE 5.4 (b)

VARIATION OF TWELFTH HARMONIC TORQUE WITH MODULATION INDEX FOR DIFFERENT MOTOR SPEEDS
(INVERTER OUTPUT VOLTAGE TO FREQUENCY RATIO IS CONSTANT)

	Mod. index	At Standstill	At 600 rpm	At 960 rpm	At 1080 rpm	At 1140 rpm	At 1176
p = 3	1.00	0.681	1.133	1.557	1.381	1.025	0.786
	0.90	0.898	1.758	3.464	4.664	5.397	5.828
	0.80	1.348	2.808	5.890	8.047	9.332	10.068
	0.70	1.751	3.501	6.964	9.233	10.566	11.334
	0.60	1.569	2.872	5.191	6.610	7.434	7.923
	0.50	0.783	1.073	1.086	0.885	0.768	0.776
	0.40	3.419	5.969	9.746	11.847	13.084	13.868
	0.30	8.179	13.547	20.981	24.997	27.379	28.916
	0.20	13.483	21.246	31.363	36.723	39.951	42.071
	0.10	16.397	24.798	35.131	40.451	43.631	45.727
	0.00	0.000	0.000	0.000	0.000	0.000	0.000
p = 6	1.00	0.681	1.133	1.557	1.381	1.025	0.786
	0.90	1.015	1.976	3.860	5.175	5.978	0.452
	0.80	1.826	3.743	7.692	10.431	12.067	13.011
	0.70	3.121	6.274	12.398	16.397	18.740	20.105
	0.60	4.929	9.507	17.695	22.716	25.629	27.360
	0.50	7.261	13.324	23.237	28.989	32.324	34.358
	0.40	10.040	17.464	28.585	34.763	38.374	40.638
	0.30	13.011	21.464	33.181	39.504	43.250	45.662
	0.20	15.588	24.526	36.171	42.345	46.052	48.490
	0.10	16.099	24.339	34.473	39.690	42.808	44.864
	0.00	0.000	0.000	0.000	0.000	0.000	0.000

(All quantities are in Newton-meter)

Continued.....

TABLE 5.4 (b) (CONTINUED):

	Mod.index	At standstill	At 600 rpm	At 960 rpm	At 1080 rpm	At 1140 rpm	At 1176 rpm
p = 9	1.00	0.681	1.133	1.557	1.381	1.025	0.786
	0.90	0.869	1.399	1.793	1.491	1.016	0.687
	0.80	1.061	1.647	1.963	1.537	0.967	0.565
	0.70	1.245	1.850	2.042	1.507	0.873	0.421
	0.60	1.397	1.976	2.007	1.389	0.727	0.256
	0.50	1.487	1.985	1.840	1.177	0.522	0.109
	0.40	1.471	1.835	1.524	0.860	0.252	0.246
	0.30	1.300	1.487	1.044	0.432	0.191	0.570
	0.20	0.931	0.914	0.413	0.368	0.758	1.096
	0.10	0.433	0.410	0.926	1.461	1.864	2.167
	0.00	0.000	0.000	0.000	0.000	0.000	0.000
p = 12	1.00	0.681	1.133	1.557	1.381	1.025	0.786
	0.90	0.779	1.257	1.625	1.379	0.996	0.757
	0.80	0.873	1.361	1.645	1.337	0.944	0.722
	0.70	0.955	1.430	1.609	1.253	0.871	0.683
	0.60	1.014	1.447	1.508	1.126	0.777	0.645
	0.50	1.031	1.392	1.335	0.955	0.670	0.617
	0.40	0.981	1.242	1.085	0.745	0.568	0.615
	0.30	0.835	0.975	0.754	0.519	0.525	0.674
	0.20	0.559	0.567	0.365	0.447	0.668	0.871
	0.10	0.121	0.098	0.583	0.955	1.225	1.425
	0.00	0.000	0.000	0.000	0.000	0.000	0.000

(All quantities are in Newton-meter)

Chapter - 6

CONCLUSIONS

A method of analysing the performance of symmetrical induction machinery with applied voltage of any periodic form has been developed. Fourier analysis provides a convenient method for studying induction motor performance when supplied from a thyristor inverter. In Chapters 3 and 4 the voltage applied to the motor is assumed to be of six step waveform. This provides a sufficiently accurate calculation of motor performance. However, in practical systems, the applied voltage is not rectangular and this method should not be used for predicting current waveforms.

The power and developed electromagnetic torque may be expressed in terms of inductance coefficients and the product of the winding currents for any form of excitation function at constant frequency (Chapter - 4) in stationary reference frame. Even instantaneous torque can be expressed in the form of an infinite series containing function of induction machine constants. This makes it possible to grasp the overall characteristics of the said torque. The nature of the electromagnetic torque produced by the motor has been studied.

A method of estimating torque pulsations under steady state using the results from single phase equivalent circuit

has been presented. The method indicates that the torque fluctuations are mainly due to the fundamental flux and harmonic currents and vice versa. Since the fundamental flux is normally 1.0 p.u. the above perturbation in torque can be reduced only by reducing the harmonic currents.

A method of determining the steady state pulsating harmonic torque acting on the rotor of the induction machine when supplied from inverter has been given. The effect of speed and change of inverter voltage and frequency have been incorporated in the analysis. It is shown that the pulsating harmonic torque has significant effect on instantaneous torque especially at high speeds. For practical systems this pulsations at low frequencies become greater than that of steady harmonic torque at close to synchronous speed region. The harmonic produced steady state torque are usually very small and can be neglected in comparison with the fundamental torque.

It has been also shown that the torque harmonics have frequencies of the type '6 $k\omega_s$' for the case of motor being fed by inverter where ω_s is the frequency of the inverter and k is an integer. As it has been shown that for computation of torque harmonics of the order of '6k' it is sufficient to consider the current harmonics of the order (6k-1) and (6k+1).

The control of current and torque harmonics by modulating the input stator voltage within the inverter has been studied in Chapter - 5. It is shown that for the balanced system, number of equal pulses per half cycle should be a multiple of three. By increasing the number of pulses per half cycle, the significant harmonic frequency can be increased which then becomes easier to filter out. That is, with pulsed inverter, the pulsations at the modulating frequency predominate. As it can be reduced by reduction in harmonics, therefore it will be advisable to keep the number of pulses per half cycle to be high enough. But in this scheme, the maximum available value of fundamental voltage decreases below that of six step waveform.

The common drawback of multiple pulse scheme lies in the lowering of the efficiency. Each commutation of the main thyristors shown in Figure 5.1(a) entails certain amount of energy losses and total loss is its multiplication by the number of switching per half cycle.

The theoretical effect of number of pulses per half cycle and modulation index on several performance features of the induction motor also have been reviewed. Certainly the ruggedness of the squirrel cage induction motor plus the ability to produce variable voltage and inverter operating frequency of the modulating scheme are valuable and unique

REFERENCES

[1] S.B. Dewan and A. Straughen. Power Semiconductor Circuits, New York : John Wiley & Sons, 1975.

[2] M. Ramamoorty, Thyristors and their Applications, New Delhi: East West Press, 1977.

[3] G. De, Principles of Thyristorised Converters, New Delhi, Oxford and IBH Publishing Co., 1982.

[4] B.R. Pelly, Thyristor Phase Controlled Converter and Cycloconverter, New York : Wiley Interscience, 1971.

[5] P.C. Krause, 'Method of Multiple Reference Frames Applied to Analysis of Symmetrical Induction Machinery', IEEE Trans. PAS, Vol. PAS-87, No.1, pp.218-227, January 1968.

[6] A.E. Fitzerald and C. Kingsley, Electric Machinery, Tokyo, Kogakusha Co.,Ltd. Japan, 1961.

[7] P.C. Krause and C.H. Thomas, 'Simulation of Symmetrical Induction Machinery', IEEE Trans., PAS, Vol. PAS-84, No. 11, pp. 1035-1053, November 1965.

[8] R.S. Ramshaw and K.R. Padiyar, 'Generalized System Model for Slip Ring Machine', Proc. IEE (London), Vol. 120, pp. 647-658, June 1973.

[9] W. Shephard, 'On the Analysis of the Three-Phase Induction Motor with Voltage Control by Thyristor Switching', IEEE IGA, Vol. IGA-4, No. 3, May/June 1968.

[10] L.J. Jacovidas, 'Analysis of Induction Motor Drives with a Nonsinusoidal Supply Voltage Using Fourier Analysis', IEEE Trans. IA, Vol.IA-9, No.6, Nov./Dec. 1973.

[11] D.A. Paice, 'Induction Motor Speed Control by Stator Voltage Control', IEEE Trans. PAS; Vol. PAS-87, No.2, February 1968.

[12] Kumo Koga, 'Instantaneous Torque of Induction Motor Driven by Nonsinusoidal Wave Power Source', Elect. Engg., in Japan, Vol. 103, No. 1, pp. 63-70, 1983.

[13] T.A. Lipo, P.C., Krause and H.E. Jordan, 'Harmonic Torque and Speed Pulsations in a Rectifier-Inverter Induction Motor Drive', IEEE Trans. PAS, Vol. PAS-88, No. 5, May 1969.

[14] E.A. Klingshirn and H.E. Jordan, 'Poly Phase Induction Motor Performance and Losses on Nonsinusoidal Voltage Sources', IEEE Trans. PAS, Vol. PAS-87,,No. 3, pp. 624-627, March 1968.

[15] J.J. Pollack, 'Advanced Pulse Width Modulated Inverter Techniques', IEEE Trans. IA, Vol. IA-8, No.2, March/April 1972.

[16] R.D. Adams, 'Several Modulation Techniques for a Pulse Width Modulated Inverter', IEEE Trans. IA, Vol. IA-8, No. 5, Sept./Oct. 1972.

[17] B. Mokrytzki, 'Pulse Width Modulated Inverters for AC Motor Drives', IEEE Trans. IGA, Vol. IGA-3, No. 6, Nov./Dec. 1967.

[18] H.S. Patel and R.G. Hoft, 'Generalized Techniques of Harmonic Elimination and Voltage Control in Thyristor Inverters: Part I Harmonic Elimination', IEEE Trans. IA, Vol. IA-9, No. 3, May/June 1973.

[19] A. Bashir, B.D. Pradhan and G.N. Revankar, 'Voltage Control in 3-phase Stepped Wave Inverters', Int. J. Electronics , May 1982.

[20] Nagasaka, Iwakane, Urano and Kai, -Nat'l, Conv., IEE, Japan, No.582, 1974.

APPENDIX A

DETAILS OF THE SQUIRREL CAGE INDUCTION MOTOR USED

Number of phases	: 3	Number of poles	: 6
Connection	: Star	Output power	: 10 h.p.
Line to line voltage	: 220 volt	Frequency	: 60 Hz

The parameters of the motor per phase are:

Stator leakage inductance	L_{11} = 0.0364801 henry
Rotor leakage inductance (referred to stator)	L'_{22} = 0.0357002 henry
Mutual inductance between stator and rotor phase (referred to stator)	M = 0.0351467 henry
Stator resistance	r_1 = 0.294 Ohms
Rotor resistance referred to stator	r'_2 = 0.144 Ohms

APPENDIX B

INVESTIGATION OF THE STEADY STATE OPERATION OF A THREE PHASE VOLTAGE SOURCE INVERTER WITH 120° CONDUCTION FOR A GIVEN R-L LOAD IN STAR CONNECTION

Assumptions:

(i) Two thyristors conduct at any instant

(ii) Each thyristor conducts for two third of the total time period

(iii) Commutation intervals are neglected

(iv) Balanced three phase load without any mutual effect.

At the end of each T/6 period, one thyristor is turned off and the corresponding load terminal is disconnected from the source. But the presence of any finite load inductance prevents any instantaneous change in the individual branch current of the load. Therefore, during the interval T/6 it is clear that two distinct circuit states exist;
(i) as phase load disconnected from the inverter and
(ii) all the load phases connected to the inverter.

For example, as mode-I begins with turning off of S_5 and turning on of S_1, i_{an} is to be equal to i_{bn} in magnitude if load terminal 'c' gets disconnected simultaneously from the corresponding output terminal of the inverter. But in the previous mode of operation, i_{cn} was positive. Therefore,

current through phase 'c' cannot be zero instantaneously at the starting of mode-I. As a result, i_{cn} is diverted into the feedback diode D_2 and closes its loop through S_6 shorting the load branch 'b' and 'c'. Thus in the beginning of mode-I, $v_{bc} = 0$ and not $- V_{dc}/2$ as in the case of resistive load and $v_{ca} = - V_{dc}$ and not $- V_{dc}/2$.

Let 't_o' be the instant at which i_{an} becomes equal to i_{bn} but of negative magnitude, then

$$i_{an}(t_o) = 2.V_{dc}(1-e^{-t_o/\tau})/3R = I' \text{ (say)} \tag{B.1}$$

where $\tau = L/R$, time constant of the load and initial current in phase 'a' is zero (as this phase was already open at the end of previous mode).

In the rest of the interval of mode-I, with the disconnection of line 'c', voltage V_{dc} is experienced by branch 'a' and 'b' in series. Therefore, at the end of mode-I

$$i_{an}(T/6) = V_{dc}/2R + e^{-(T-6\,t_o)/6\tau}\{I' - V_{dc}/2R\} = I'' \text{ (say)} \tag{B.2}$$

As mode-II begins with turning off of S_6 and turning on of S_2, phase 'c' continues to conduct for a while through D_3, so that the branch 'a' and 'b' becomes shorted, enabling the magnitude of i_{bn} to come down to zero value and i_{cn} to become negative. Thus in the initial part of mode-II, i_{bn} plays the role of i_{cn} of mode-I though in the reverse direction.

Therefore,

$$i_{an}(T/6 + t_o) = V_{dc}(1-e^{t_o/\tau})/3R + I'' e^{-t_o/\tau} = I''' \text{ (say)} \qquad (B.3)$$

The sub-interval of mode-II will end again at t_o when i_{an} becomes equal to i_{cn} of negative magnitude and D_3 ceases to conduct. Consequently,

$$i_{an}(T/3) = V_{dc} \{1-e^{-(T-6\ t_o)/6\tau}\}/2R + I''' e^{-(T-6\ t_o)/6\tau}$$
$$= I'''' \text{ (say)} \qquad (B.4)$$

But in the steady state owing to the half cycle as well as the phase symmetry

$$i_{an}(T/3+t_o) = - V_{dc}(1-e^{-t_o/\tau})/3R + T'''' e^{-t_o/\tau} = 0 \qquad (B.5)$$

Now combining equation (B.4) and (B.5)

$$(1-e^{-t_o/\tau})/3 = (e^{-t_o/\tau} - e^{-T/6\tau})/2 + \frac{RI'''}{V_{dc}} e^{-T/6\tau} \qquad (B.6)$$

with the help of equation (B.3) the aforesaid equation becomes

$$(1-e^{-t_o/\tau})/3 = (X + Y) + \frac{RI''}{V_{dc}} e^{-(T+6\ t_o)/6\tau} \qquad (B.7)$$

where,

$$X = (e^{-t_o/\tau} - e^{-T/6\tau})/2$$

and

$$Y = \{e^{-T/6\tau} - e^{-(T+6t_o)/6\tau}\}/3$$

Again combining equations (B.7) and (B.2)

$$(1-e^{-t_o/\tau})/3 = (X + Y + Z) = \frac{RI'}{V_{dc}} e^{-T/3\tau} \qquad (B.8)$$

where,

$$Z = \{e^{-(T+6\ t_o)/6\tau} - e^{-T/3\tau}\}/2$$

Substituting the expression of I' from equation (B.1) in the above equation, $2(1-e^{-t_o/\tau})(1 - e^{-T/6\tau} - 2\,e^{-T/3\tau})$

$$= 3(1+ e^{-T/6\tau})\ (e^{-t_o/\tau} - e^{-T/6\tau}) \qquad (B.9)$$

$$\text{Therefore } t_o = -\tau.\ \log_e \{\frac{U + V.e^{-T/6\tau}}{U + V}\} \qquad (B.10)$$

where

$$U = 2\ (1-e^{-T/6\tau} - 2e^{-T/3\tau})$$

and

$$V = 3\ (1 + e^{-T/6\tau})\ .$$

Figure B.1 shows not only the line to line and line to load neutral voltages but also illustrates the thyrister and diode conducting sequence along with the load current flow in the inverter for the assumed current condition. Evidently the voltage waveform is different from the case of pure resistive load which was shown in Fig. 2.5. It can be noted that open circuit occur twice per phase per cycle provided the current intersects the zero axis. Since the inverter is triggered symmetrically, it is clear that three load

currents are both three phase and half wave symmetric. Hence, the zero current interval and phase angle between voltage and current-zero crossings are identical in all three phases.

Further 't_o' changes with τ, i.e. the power factor of the load. Because of the load dependent nature and consequent uncertainity of the output voltage the inverter with 120° conduction schedule has only limited application as a voltage source inverter.

COMPARISON OF 180° CONDUCTION (3 SCRs) CONTROL AND 120° CONDUCTION (2 SCRs) CONTROL IN VOLTAGE SOURCE INVERTER

Sl. No.	180° conduction control scheme	120° conduction control scheme
1.	Number of thyristor conduct at a time is three	Number of thyristor conduct at a time is
2.	Conduction period of each thyristor is 180°	Conduction period of each thyristor is 12
3.	All terminals of the load are connected with the input via 3 thyristors in each interval, may be two thyristors in the upper half with positive bus and rest is with negative bus or vice versa. So voltage waveforms are defined at all intervals	Two of the load terminals get connected d.c. supply and third load terminals is floating. The potential of floating terminal at any instant will depend on the nature of the load and will not remain constant during the duration of the interv So voltage level is not defined.
4.	Turning on of any thyristor in the series branch $(S_1,S_4)(S_3,S_6)$ and (S_5,S_2) is followed immediately after the commutation of the thyristors in that branch. This enough time may not be provided for the commutation of the thyristors and two thyristors in series may simultaneously conduct resulting into short circuit of the source by these thyristors.	For each thyristor there is an interval of $\pi/3$ between the commutation and turning on the series thyristors. This enough time is made available for the outgoing thyrist to commutate before the thyristor in serie turned on. Therefore commutation is relia and possibility of the series thyristors conducting simultaneously is much less.
5.	Utility factor = P_o/P_T = 0.33	Utility factor P_o/P_T = 0.289

NOTE : P_o : rated output power of the inverter; P_T : measure of total power handling capability of the thyristors employed in the inverter and is equal to $(N.V_{drm}.I_{rms})$ where n is the number of thyristors, V_{drm} is repetitive peak forward voltage and I_{rms} is the rated r.m.s. forward current.

APPENDIX C

VARIOUS CONTROL SCHEMES FOR SOLID STATE SPEED CONTROL OF INDUCTION MOTOR

Voltage control location	Input Voltage			Input Rectifier		Intermediate	Inverter charts.			Output control
	Fixed D.C.	Fixed A.C.	Trans-former	Diode	S.C.R.	D.C. Chopper controlled	Fixed Bridge	Phase shif-ting bridge	High freq. chopp-er	Variable voltage transfor-mer
At the input			*	*			*			
		*			*		*			
At the adjusta-ble bus		*		*		*	*			
	*					*	*			
Within the inverter		*		*				*		
		*		*					*	
At the output		*		*			*			*
		*		*			*			